Material selected by
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O1 - Open on-line course topic "Maths in Finance"

Timişoara
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Co-funded by ERASMUS+ Program of the European Union, Key Action 2Project: 2016-1-RO01-KA201-024518, "Discover Real Everywhere Applications of Maths – DREAM"
Beneficiary: Colegiul Național "Constantin Diaconovici Loga", Timișoara

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Maths in Finance

Contents

Foreword

This intellectual output was create in the Erasmus project " DREAM - Discover Real Everywhere Applications of Maths", identification number: 2016-1-RO01-KA201-024518, through the collaboration of students and teachers from Colegiul Național "Constantin Diaconovici Loga" Timișoara, Romania, 1o Geniko Lykeio, Aigiou, Greece, Agrupamento de Escolas Soares Basto, Oliveira de Azeméis Norte, Portugalia and "TIBISCUS" University of Timișoara, Computers and Applied Computer Science Faculty.

The project main objective was to build up a new maths teaching/learning methodology based on real-life problems and investigations (open-ended math situations), designed by students and teachers together. The activities involved experimentations, hands-on approach, outdoor activities and virtual and mobile software applications. The developed material was transform into on-line courses and is freely available to all interested communities, in order to produce collaborative learning activities.

O1 - Maths in Finance has the purpose to facilitate the understanding of the usefulness of some mathematical chapters that are applicable to financial life.

The activities in this pack feed into the Skills and Capability Framework by providing contexts for the development of Thinking, Problem Solving and Decision Making Skills and Managing Information. Open-ended questions facilitate pupils to use Mathematics. ICT opportunities are provided through using Moodle platform and additional tasks researching information using the internet.

This intellectual output comprises five lesson scenarios and guides the teacher in creating interactive and exciting lessons.

Introduction

The importance of financial capability

Financial capability is an essential life skill in a society where individuals are faced by increasingly complex financial challenges. A skill depends on using mathematics in everyday work and life situations and is an important preparation for adult life.
Young people face increasingly complex financial decisions: an increasing range of non-cash methods of payment; the easy availability of credit; dealing with student debt; and a flexible job market where the idea of one job for life is diminishing.

Financially capable citizens are more likely to make informed decisions leading to financial freedom. They are less likely to get into financial difficulties and have to fall back onto government benefits. As competent and confident consumers, they can make the most of the available financial products and engage more fully with the financial services industry.

Opportunities exist to develop the Key Elements of:

- Economic Awareness – applying mathematical skills in everyday financial planning and decision-making.
- Employability – exploring how the skills developed in mathematics will be useful for business records; demonstrating how to be enterprising when discussing potential fund raising activities.
- Citizenship – developing the capacity of young people to make informed and responsible decisions.
- Moral Character – demonstrating an ability and willingness to develop logical arguments.

Methodology

Learning and teaching in mathematics can be made more effective where a balance of practical, oral and written tasks is provided. This pack provides information and scenarios to assist in this task. The intention is to provide young people five activities that are related to their age and attainment. One aspect of the pack is the use of the PowerPoint presentations or educational videos in order to stimulate whole-class discussions before and after the activities have been completed. The emphasis should be on helping young

people understand what the problems are and to become aware of the technical vocabulary surrounding the issues..

General Pedagogical Recommendations:

- Watching a power point presentation or a film which introduces the theme of real-life lesson
- Discovering the link between real life and the mathematical concept that governs the given situation
- Recall theoretical mathematical concepts
- Frontal discussion of the real situation in the matter
- Solving some parts of the problem by group of students using mathematical tools: minicomputers, Geogebra, Excel, internet
- Discussing solutions, looking for the optimal option
- Student's task: loads the optimal solution found on the MOODLE platform
- Teacher's task: controls the homework of the student and provides a feedback.

Examples from O1 - Maths in Finance use the notions and the properties of following chapters:

- Financial Mathematics: Percentages, Interests;

- Exponents and logarithms;

- Quadratic functions;

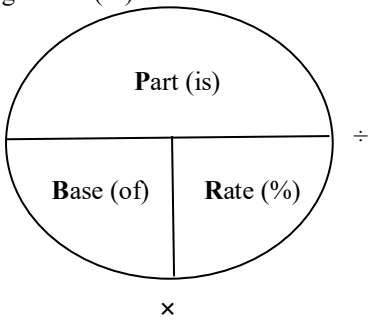
- Finding Maxima and Minima using second derivative.

Theoretical background

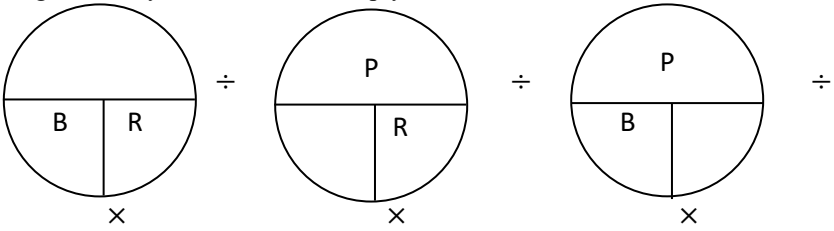
- Financial Mathematics: Percentages, Interests, Statistic

The Percent Formula

Percentages involve three quantities: the **B**ase, the **P**art and the **R**ate.
Base (**B**) - the total or whole (of)
Part (**P**) - a part of the total that is being compared to it (is)
Rate (**%**) - the percentage rate (%)



The above diagram presents a formula that may be used to solve for any type of percentage problem. It shows an easy way to remember how to find any one of the missing terms. Simply put your finger over the missing term and look at the diagram to see the relationship between the other two terms. The diagram tells you whether to multiply or divide.



a. b. c.

a. When the **P**art is missing multiply the base and rate
b. When the **B**ase is missing divide the part by the rate
c. When the **R**ate is missing divide the part by the base

a. Find 4% of 245.
4% x 245 = P
0.04 x 245 = P
9.8 = P
b. 8% of what number is 74

$$\frac{74}{0.08} = B$$

925 = B
c. What percent of 96 is 36?

$$\frac{36}{96} = R$$

0.375 = R so, R = 37.5%

Tasks

1. In part because of the popularity of outlet malls, about 300 of the 1800 traditional retail malls across the United States may close in the next few years. Find the percent of retail malls that may close?
2. The bee hummingbird is the smallest bird. It can weigh as little as 2 grams. The largest bird, the ostrich, can weigh as much as 150 kilograms. What percent of the weight of a bee hummingbird is the weight of an ostrich?
3. The U.S. Postal Service handles 170,000,000,000 pieces of mail each year. This is 40% of the world's total. How many pieces of mail are sent ach year?
4. A bike is on sale for $170. This is 80% of the regular price. Find the regular price.
5. During a flu epidemic, 146 students out of the 680 who attend Lincoln Middle School were absent. What percent were absent?

Simple interest

Simple interest is a fixed percentage (r %) of the principal (Po) paid to an investor each year irrespective of the number of years (n) the principal has been left on deposit. However, in modern business this approach is rarely adopted – the interest is compounded, that is,

$$S = Po(1 + r\%n)$$

Compound interest

Compound interest is the foundation of financial and investment mathematics. In particular, methods are developed for calculating the accumulated value and present value of an investment. If a person deposits Po with a financial institution at a rate of interest of $r\%$ per annum and leaves any interest to accumulate within the account – the interest is earning interest.

After t years the initial investment grows to $Po(1 + r)^t$. After t intervals of time, where t can be a month, quarter year, half year, etc, the accumulated value is

$$S = Po(1 + r)^t$$

The interest earned grows, because the amount of money it is applied to grows with each payment of interest. We earn not only interest, but interest on the interest already paid.

Annual Percentage Rate (APR)

Annual Percentage Rate (APR): Rates of interest only have meaning when they are related to a time interval. Rates of interest, expressed above, giving an actual rate of interest over a stated interval of time, are effective rates of interest. Where the effective rate of interest is expressed as a fraction of a year $(1/p)$ it may be converted to an annual rate by multiplying by p. thus, 3% per quarter would be quoted as '12% per annum, converted quarterly'. Interest rates quoted in this way are known as nominal rates of interest. Quoted interest rates on savings products offered by financial institutions are often nominal rates, e.g. converted half yearly. Corresponding to a nominal rate of interest, there exists an effective annual rate of interest.

If a person invests €100 for one year at 10% per annum, convertible half-yearly, (effective rate of interest of 5% per half year) – not the same as an effective rate of interest of 10% pa - the amount at the end of the year is $€100\ (I + \frac{0.10}{2}) \cdot 2 = 1.1025.$

If the annual rate of interest is i% pa the amount at the end of the year is $€100\ (I + r)$. Therefore, $1 + r = 1.1025$, giving $r = 10.25\%$. A rate of interest expressed as 10% pa, convertible half yearly, is the same as an

effective rate of interest of 10.25% or quoted as the APR (annual percentage rate).

This process may be generalized as follows:
Nominal rate compounded n times per year: $S = Po(1 + \frac{r}{n})^{nt}$
APR rate compounded annually: $S = Po(1 + APR)^t$
Since the yield is the same, $Po(1 + \frac{r}{n})^{nt} = Po(1 + APR)^t$
giving an APR of $(1 + \frac{r}{n})^n - 1$.

Exemples for Compound Interest

1. Invest €500 that earns 10% interest each year for 3 years, where each interest payment is reinvested at the same rate:

Table 1 Development of Nominal Payments and the Terminal Value, *S*.

	Nominal Interest	*S*
Year 1	50	550 = (1.1)
Year 2	55	605 = 500(1.1)(1.1)
Year 3	60.5	665.5 = $500(1.1)^3$

2. A principal of €25000 is invested at 12% interest compounded annually. After how many years will it have exceeded €250000?

$10P = P \cdot (1 + r)^n$
$250{,}000 = 25{,}000 \cdot (1.12)^n$
$10 = 1.12^n$
$\ln 10 = n \cdot ln1.12$
$\frac{ln10}{ln1.12} = n \approx 20.3177$

Compounding can take place several times in a year, e.g. quarterly, monthly, weekly, continuously. This does not mean that the quoted interest rate is paid out that number of times a year!

3. Assume the €500 is invested for 3 years, at 10%, but now we compound quarterly:

Table 2 Quarterly Progression of Interest Earned and End-of-Quarter Value, *S*.

Quarter	Interest Earned	*S*
1	12.5	512.5
2	12.8125	525.3125
3	13.1328	538.445
4	13.4611	551.91

Generally: $S = P \cdot \left(1 + \frac{r}{m}\right)^{n\,m}$ where *m* is the amount of compounding per period *n*.

4. €10 invested at 12% interest for two years. What is the future value if compounded
a) annually ? b) semi-annually ? c) quarterly?
d) monthly ? e) weekly?

As the interval of compounding shrinks, i.e. it becomes more frequent, the interest earned grows. However, the increases become smaller as we increase the frequency. As compounding increases to continuous compounding our formula converges to:

$$S = P \cdot e^{rt}$$

5. A principal of €10000 is invested at one of the following banks:
a) at 4.75% interest, compounded annually
b) at 4.7% interest, compounded semi-annually
c) at 4.65% interest, compounded quarterly
d) at 4.6% interest, compounded continuously

Which is the best bank to lodge the money?
=>
a) $10{,}000(1.0475) = 10{,}475$
b) $10{,}000(1+0.047/2)^2 \approx 10{,}475.52$
c) $10{,}000(1+0.0465/4)^4 \approx 10{,}473.17$
d) $10{,}000e^{0.046t} \approx 10{,}470.74$

6. Determine the annual percentage rate, APR, of interest of a deposit account which has a (simple) nominal rate of 8% compounded monthly.

$$\left(1+\frac{0.08}{12}\right)^{1\cdot 12} \approx 1.083$$

7. A firm decides to increase output at a constant rate from its current level of €50000 to €60000 over the next 5 years. Calculate the annual rate of growth required to achieve this growth.

$50000(1+r)^5 = 60000$

$(1+r)^5 = 1.2$

$1+r = \sqrt[5]{1.2}$

$r \approx 3.7\%$

Statistic

Statistics is a branch of applied mathematics concerned with collecting, organizing, and interpreting data. The data are represented by means of graphs. Statistics is also the mathematical study of the likelihood and probability of events occurring based on known quantitative data or a collection of data. Statistics, thus attempts to infer the properties of a large collection of data from inspection of a sample of the collection thereby allowing educated guesses to be made with a minimum of expense.

There are three kinds of averages commonly used in statistics. They are: mean, median, and mode

The mean of a set of values is the sum of the values divided by the number of values. It is also called the average.

The median refers to the midpoint in a series of numbers. To find the median, arrange the numbers in order from smallest to largest. If there is an odd number of values, the middle value is the median. If there is an even number of values, the average of the two middle values is the median.

The mode of a set of values is the value that occurs most often. A set of values may have more than one mode or no mode.

- Exponents and logarithms

Exponents

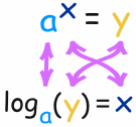
The **exponent** of a number says **how many times** to use the number in a multiplication.

Exponents have the following properties:

1. If n is a positive integer and b is any real number then $b^n = \underbrace{b \times b \ldots\ldots \times b}_{n\ factors}$
2. $b^{\frac{1}{n}} = \sqrt[n]{b}$, and if n is even we take this to mean the positive nth root of b.
3. If b ≠ 0 then b^0 = 1. b^0 is undefined for b = 0.
4. If p and q are integers then $b^{\frac{p}{q}} = \left(b^{\frac{1}{q}}\right)^p = (b^p)^{\frac{1}{q}}$
5. $b^x \times b^y = b^{x+y}$ whenever both sides of this equation are defined.
6. $\frac{b^x}{b^y} = b^{x-y}$ whenever both sides of this equation are defined.
7. $b^{-x} = \frac{1}{b^x}$ whenever both sides of this equation are defined.
8. $(ab)^x = a^x b^x$ whenever both sides of this equation are defined.
9. $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$ whenever both sides of this equation are defined.

Logatithms

Logarithm is the exponent to which a base is raised to obtain a given number.



For any real number $b > 1$ and any $x > 0$, $\log_b x$ is equal to that number to which b must be raised to obtain the number x. One can think of $\log_b x$ as the answer to the question $b^? = x$. The number $\log_b x$ is called the logarithm to base b of x.

The function $\log_b x$ satisfies the following rules:

Rule 1: For any real number x, $\log_b b^x = x$

Rule 2: For any real number $x > 0$, $b^{\log_b x} = x$

Rule 3: For any real numbers $x > 0$ and $y > 0$, $\log_b xy = \log_b x + \log_b y$

Rule 4: For any real numbers $x > 0$ and $y > 0$, $\log_b \frac{x}{y} = \log_b x - \log_b y$

Rule 5: For real numbers x and n, with $x > 0$, $\log_b x^n = n \log_b x$

Rule 6: For numbers $x > 0$, $a > 1$ and $b > 1$, $\log_b x = \log_b a \times \log_a x$.

Logarithms to base 10 are in common use and for this reason they are called Common Logarithms.
Logarithms to base *e* are of special importance. They are often called natural logarithms or Napierian logarithms, and the symbol *ln x* is often used for them. Thus $ln\ x = log_e\ x$. Any exponential function may be written in the form e^{kx}, where the constant *k* may be negative.

Exponential Form	Logarithmic form
$3^2 = 9$	$\log_3 9 = 2$
$4^y = x$	$\log_4 x = y$
$a^b = c$	$\log_a c = b$
$5^{1/2} = p$	$\log_5 p = 1/2$
$m^n = k$	$\log_m k = n$
$4^3 = 64$	$\log_4 64 = 3$
$x^0 = 1$	$\log_x 1 = 0$
$a^1 = a$	$\log_a a = 1$

[source: http://www.math-for-all-grades.com]

- Quadratic functions

Quadratic function: function whose equation is a polynomial of the second degree, i.e. has the form $y = ax^2 + bx + c$, where $a \neq 0$

Quadratic equations $ax^2 + bx + c = 0$

a. discriminant is the number $d = b^2 - 4ac$

b. if $d > 0$: two solutions $x_1 = \dfrac{-b + \sqrt{d}}{2a}$, $x_2 = \dfrac{-b - \sqrt{d}}{2a}$

c. if $d = 0$: one solution (also called two coinciding solutions) $x_1 = x_2 = \frac{-b}{2a}$

d. if $d < 0$: solutions (in fact, the solutions are non-real complex numbers)

e. the solutions of the equation are the zeroes of the corresponding quadratic function

Graph of a quadratic function: parabola

f. sign of a determines whether parabola has opening upwards ($a > 0$) or downwards ($a < 0$)

g. sign of d determines the number of x-intercepts

i. if $d > 0$: x-axis and parabola intersect in two points

ii. if $d = 0$: x-axis and parabola have one common point, horizontal axis is tangent to parabola

iii. if $d < 0$: x-axis and parabola do not intersect

h. c is the y-intercept of the parabola

Vertex of a parabola

i. is the highest/lowest point of the parabola

j. y-value of the vertex is the maximum/minimum value of the corresponding quadratic function

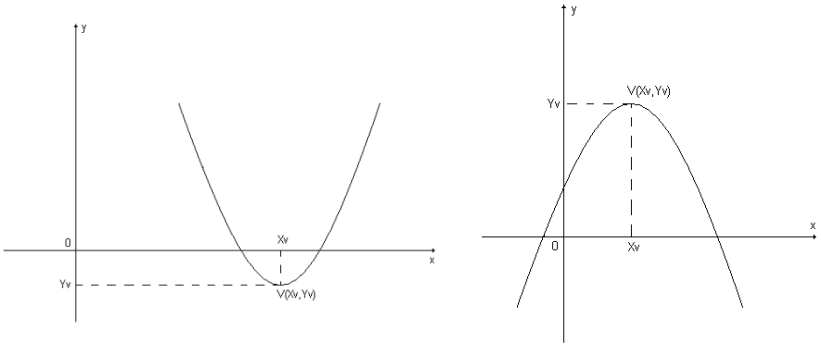
k. x-coordinate of the vertex is the x-value whose corresponding y-value is maximum/minimum value

l. x-coordinate of the vertex is $\frac{-b}{2a}$

m. y-coordinate of the vertex is found by plugging its x-coordinate into the equation, $y = \frac{-d}{4a}$

If $a > 0, V(\frac{-b}{2a}; \frac{-d}{4a})$ is the minimum.

If $a < 0, V(\frac{-b}{2a}; \frac{-d}{4a})$ is the maximum.



- Finding Maxima and Minima using second derivative

Find the stationary points and identify whether these are maximum, minimum or inflection points.

1. $y = -x^2 - 5x - 5$,

First order condition: slope of function = 0 at stationary point

$\frac{dy}{dx} = -2x - 5 = 0$, now solve for the value of x at the stationary point:

$\Rightarrow \quad x = -\frac{5}{2}$ or –2.5.

Substitute in this value to the original function to find the value of y. The value of y at this point is $y = -(-\frac{5}{2})^2 - 5(-\frac{5}{2}) - 5 = 1.25$. Thus, stationary point at (-2.5, -1.25)

Second Order Condition: check the sign of second derivative at this point. To find if this is a max or min point, we need to evaluate the change in the slope at x = -2.5.

Thus, we check the sign of the second derivative $\frac{d^2y}{dx^2} = -2$, which is negative when $x = -\frac{5}{2}$ (and in fact, for all values of x).

Thus, the change in the slope of the function is negative around the stationary point, which indicates a maximum.
So (-2.5, -1.25) is a local maximum of y.

2. $y = 2x^2 + 4$

First order condition: slope of function = 0 at stationary point

$$\frac{dy}{dx} = 4x = 0$$

$$x = 0$$

The value of y at this point is y = 0 +4 = 4. Thus, stationary point at (0, 4)
Second Order Condition: check the sign of second derivative at this point.
To find if this is a max or min point, we need to evaluate the change in the slope at x = 0
Thus, we check the sign of the second derivative

$\frac{d^2y}{dx^2} = 4 > 0$ at x=0 (and at all values of x).

So we have a minimum at (0,4).

The optimization problems involve a process of maximizing and minimizing functions.

1. The cheapest cities in Europe

Field of application: Financial Mathematics
Required knowledge: Arithmetic mean, Percentages, Average, Comparison, , Weightings , Statistics.
Project: Timisoara, Patras or Porto is the best financial place for a first appointed high school teacher to live ?
Moodle: http://srv-1lyk-aigiou.ach.sch.gr/moodle/course/view.php?id=2&sesskey=NG9hBRBq7T#section-1
Authors: Georgios Kottas, Andreas Bodiotis, students from 1o Geniko Lykeio, Aigiou, Greece
Coordinator: Nikolaos Diamantopoulos
The assignment of this lesson is to submit 3 files detailing your conclusions about where is better financially for a first appointed high school teacher with no family to live, Timisoara/Romany, Patras/Greece or Porto/Portugal?
To help yourself, you can read the following information.
(Source: https://www.worldwidecostofliving.com/asp/wcol_HelpIndexCalc.asp)
Resources: Information sheet - Source: EIU world wide cost of living File.
Generalization: Research can be extended to any city in Europe and changing living conditions.

Timisoara, Patra or Porto is the best financial place for a first appointed high school teacher to live? (Project)

To facilitate your calculations make the following assumptions:

1) Suppose that 35% of base salary is the spendable income,ie the assumed amount that the employee ordinarily spends on food, clothing, recreation and transportation etc.
2) There is no family.
3) The housing cost consists of renting a one person apartment.

What is the cost of living index?

The cost-of-living index shows the difference in living costs between cities. The cost of living in the base city is always expressed as 100. The cost of living in the destination is then indexed against this number. So to take a simple example, if Athens is the base (100) and Rome is the destination, and the Rome index is 120, then Rome is 20% more expensive than Athens. Similarly, if Bucharest is the base, Porto is the destination, and the Bucharest index is 80, than the cost of living in Bucharest is 80% of Porto's.

What is the methodology behind the index?

The cost-of-living index expresses the difference in the cost of living between any two cities in the survey. How is this index calculated?

Using exactly the same price data, but different methods of calculation, a number of different people could come up with a number of markedly different indices. The challenge, therefore, when seeking to construct an index is to know which method is best for the problem at hand and to represent equitably (in one figure) the general trend of price differences in separate locations. To illustrate this point, let us take a simple price survey comparing two fictional cities, "City-A" and "City-B."

	City-A	City-B
Bread (1kg)	1.00	1.25
Potatoes (1kg)	3.00	2.00
Coffee (1kg)	2.50	1.75
Sugar (1kg)	1.00	1.75
TOTAL	**7.50**	**6.75**

Assuming we give equal weight to each of the products, which of the two towns deserves the higher cost of living index number? The answer is: it all depends on how the calculation is made.
1) City-A is more expensive if we simply add up the prices of the four items in the index and compare the two cities on that basis.
2) City-B, however, is more expensive when we use City-A as a base city and calculate an index based on the average of relative prices in the two cities:

	City-A	City-B
Bread	100	125
Potatoes	100	67
Coffee	100	70
Sugar	100	175
Index	**100**	**109**

However, if the same calculation is done with City-B serving as a base city, City-A becomes the more expensive city:

	City-B	City-A
Bread	100	80
Potatoes	100	150
Coffee	100	143
Sugar	100	57
Index	**100**	**107.50**

Thus with the standard price-relatives calculation we can end up in the paradoxical situation where each city is more expensive than the other can.

Homework: Student have to upload to Moodle platform a Statistical study.

2. Biggest box problem - Cubic functions

Field of application: Financial Mathematics
Required knowledge: Cubic function, Derivate of function, Volume of the cube, Optimization.
Project: Biggest box problem
Moodle: http://srv-1lyk-aigiou.ach.sch.gr/moodle/mod/assign/view.php?id=4
Authors: Evangelia Karatza, Dimitra Skoura, students from 1o Geniko Lykeio, Aigiou, Greece
Coordinator: Spyridon Potamitis
The assignment: Suppose we intend to make an open-top box using a square piece of card of side S = 60 cm by cutting a square (of side, say *x*) from each corner of the card (see *Figure 1*). The resulting piece is then folded to form the box.

- Use the First and Second Derivative Tests to find absolute maximum and minimum values of a function.
- Use the First and Second Derivative Tests to solve optimization applications.

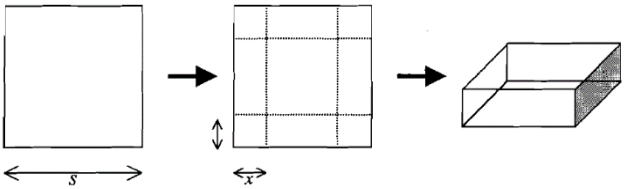


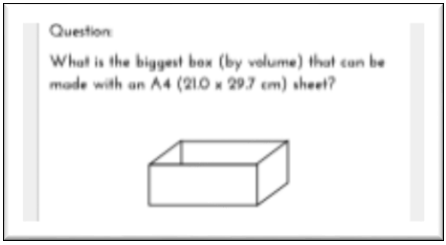
Figure 1.

Resources: https://www.youtube.com/watch?v=_oS_LjKse38
https://www.youtube.com/watch?v=UvTYc5Wqu8w

Generalization:

Research can be extended to finding the maximum volume when we use a rectangular piece of paper with different sizes.

Build the box using cardboard!



Optimization Practice

1. Two numbers add up to 40. Find the numbers and maximize their product.
2. A rectangle has a perimeter of 80 feet. What length and width should it have so that its area is a maximum? What is the maximum area?
3. An open box is to be made from a piece of metal 16 by 30 inches by cutting out squares of equal size from the corners and bending up the sides. What size square should be cut to create a box with greatest volume? What is the maximum volume?
4. Find the dimensions of the largest area rectangle that can be inscribed in a circle of radius 4 inches.
5. A 6 oz. can of Friskies cat food contains a volume of approximately 14.5 cubic inches. How should the can be constructed so that the material made to make the can is a minimum?
6. Find two numbers whose sum is 10 for which the sum of their squares in a minimum.
7. Find nonnegative numbers x and y whose sum is 75 and for which the value of xy^2 is as large as possible.
8. A ball is thrown straight up in the air. Its height after t seconds is given by $s(t) = -16t^2 + 50t$. When does the ball reach its maximum height? What is its maximum height?
9. A farmer has 2000 feet of fencing to enclose a pasture area. The field will be in the shape of a rectangle and will be placed against a river where there is no fencing needed. What dimensions of the field will give the largest area?
10. A fisheries biologist is stocking fish in a lake. She knows that when there are n fish per unit of water, the average weight of each fish will be $W(n) = 500 - 20n$ grams. What is the value of n that will maximize the total fish weight after one season? (Hint: Total Weight = number of fish, average weight of a fish)

Answers: **1.** 20 & 20; **2.** l = 20 ft, w = 20 ft, A = 400 ft^2 ; **3.** $\frac{10}{3}$ in by $\frac{10}{3}$ in, V = 725.93 in^3 ; **4.** $4\sqrt{2}$in. by $4\sqrt{2}$ in; **5.** h $\approx$ 2.65 in, r $\approx$1.32 in; **6.** 5 & 5; **7.** 50 & 25; **8.** after $1\frac{9}{16}$ sec; $\frac{625}{16}$ ft; **9.** 500 ft by 1000 ft; **10.** 12.5

3. The rules that govern investing and borrowing money

Field of application: Financial Mathematics
Required knowledge: Calculations with real numbers, Percentages, Average, Comparison, Exponents and Logarithms.
Project: The rules that govern investing and borrowing money
Moodle: http://srv-1lyk-aigiou.ach.sch.gr/moodle/course/view.php?id=2&sesskey=lEFIJrzxrR#section-5
Authors: Lepa Oana, Balica Elena, Colegiul National "C.D. Loga" Timisoara
Coordinator: Iotcovici Luminita, Romania
The assignment: Finding out, which is the most advantageous investment of a person who has a sum of money.
Research on the Internet: Find 1-3 objects, which could have been bought in 2010 with 40000 RON and which worth more money in 2017.
Resources: 6 different Assignment, Table with the interest rate at different banks for a period of 7 years, the currency exchange rate over a 10-year period
Generalization: The research can be expanded by studying real estate investment on the internet, investing in the stock market.

QUIZ

1. Cornerstone Bank pays interest of 3.5 percent compounded annually. Uptown Bank pays 3.75 percent simple interest. Which one of the following statements is true if you invest $1,000 in each bank for five years?

a. Cornerstone Bank will pay you a total of $176.59 in interest over the five years.

b. Uptown Bank will pay you $15.30 less interest over the five years than Cornerstone Bank will.

c. Cornerstone Bank will pay you a total of $175.00 in interest over the five years.

d. Uptown Bank will pay you $0.19 less interest than Cornerstone Bank over the five years.

2. Rosa wants to have $50,000 in her investment account fifteen years from now. How much does she have to deposit today to achieve her goal if she can earn 9.5 percent compounded annually?

a. $10,218.47 b. $12,524.76 c. $12,816.17 d. $13,726.90

3. Carlos has $2,413 saved today. He wants to buy a different vehicle as soon as he has $2,700 saved. How long does he have to wait to get his vehicle if he earns 4.5 percent compounded annually?

a. 2.55 years b. 2.67 years c. 2.78 years d. 3.09 years

4. ABC Co. currently pays an annual dividend of $2.60 per share. At what rate must the company increase the dividend if they want to pay $3.20 a share four years from now?

a. 5.27 percent b. 5.33 percent c. 5.41 percent d. 5.54 percent

5. Sun borrows $13,500 today at 7.90 percent compounded annually. The terms of the loan require him to repay the principal and interest in one lump sum three years from today. How much will he have to pay in three years?

a. $16,528.28 b. $16,666.67 c. $16,874.16 d. $16,958.92

6. All else equal, the present value _____ as the period increases.

a. increases b. decreases c. remains constant

7. You have been offered a business opportunity that will pay you $57,000 in six years if you invest $25,000 today. What is the expected rate of return on this investment?

a. 14.72 percent b. 15.36 percent c. 15.78 percent d. 16.22 percent

8. According to the Rule of 72, how long will it take to double your money if you can earn a 12 percent rate of return?

a. 4 years b. 6 years c. 7.2 years d. 8 years

9. Five years ago, Leslie had $23,460 in her savings account. Today, she deposited an additional $6,000. She plans to deposit another $6,000 into this account next year. How much money will she have in her account ten years from today if she earns 7.5 percent on her savings?

a. $39,560.96 b. $73,084.18 c. $93,284.88 d. $104,921.79

10. One hundred Years ago, your great-grandfather purchased a 300-acre farm for the princely sum of $2,500. Today, that farm is still owned by your family and is currently valued at $1.8 million. What rate of return has your family on this investment by your great-grandfather?

a. 5.72 percent b. 6.80 percent c. 7.03 percent d. 7.47 percent

Answers

1. d Cornerstone Bank: $\$1{,}000(1.035)^5 - \$1{,}000 = \$187.69$ compound interest
   Uptown Bank: $\$1{,}000(.0375)(5) = \$187.50$ simple interest
   Difference in interest payments = $\$187.69 - \$187.50 = \$.19$
   Uptown Bank will pay you $.19 less interest than Cornerstone Bank over the five years.

2. c $PV = \$50{,}000[1/(1.095)^{15}] = \$12{,}816.17$

	N	I/Y	PV	PMT	FV
Enter	15	9.5			50,000
Solve for			-12,816.17		

3. a $\$2{,}700 = \$2{,}413(1.045)^t$
   $1.11894 = 1.045^t$
   $\ln 1.11894 = t(\ln 1.045)$
   $.11238 = .04402\ t$
   $t = 2.55$

	N	I/Y	PV	PMT	FV
Enter		4.5	±2,413		2,700
Solve for	2.55				

4. b

	N	I/Y	PV	PMT	FV
Enter	4		±2.60		3.20
Solve for		5.33			

Manual check: $\$2.60(1.0533)^4 = \$3.20$

5. d FV = $13,500(1.0790)^3 = $16,958.92

	N	I/Y	PV	PMT	FV
Enter	3	7.90	13,500		
Solve for					-16,958.92

6. b All else equal, the present value will decrease as the period of time increases.

7. a

	N	I/Y	PV	PMT	FV
Enter	6		±25,000		57,000
Solve for		14.724403			

Manual check: $25,000(1.14724403)^6 = $57,000
Answer is 14.72% (rounded)

8. b According to the Rule of 72, it will take 72 / 12 = 6 years to double your money.

9. c FV = $23,460(1.075)^15 + $6,000(1.075)^10 + $6,000(1.075)^9 = $69,415.26 + $12,366.19 + $11,503.43 = $93,284.88

	N	I/Y	PV	PMT	FV
Enter	15	7.5	±23,460		
Solve for					69,415.26
Enter	10	7.5	±6,000		
Solve for					12,366.19
Enter	9	7.5	±6,000		
Solve for					11,503.43

Total FV = $69,415.26 + $12,366.19 + $11,503.43 = $93,284.88

10. b

	N	I/Y	PV	PMT	FV
Enter	100		±2,500		1,800,000
Solve for		6.8005096			

Manual check: $2,500(1.068005096)^100 = $1,800,000
Answer is 6.80% (rounded)

4. The bank loan and the rates

Field of application: Financial Mathematics
Required knowledge: Calculations with real numbers, Percentages, Average, Comparison, Exponents and Logarithms.
Project: Fixed rate vs Variable rate
Moodle: http://srv-1lyk-aigiou.ach.sch.gr/moodle/mod/assign/view.php?id=19
Authors: Lepa Oana, Balica Elena, Colegiul National "C.D. Loga" Timisoara
Coordinator: Iotcovici Luminita, Romania
The problem: A man wants to buy a car, but he realizes that his savings do not add up properly. Being in need of the car ASAP, he decides to get a loan from the bank. He decides upon the sum of 10000€ with an interest of 30% per year, which will be repayed in 12 rates in the course of a year.
To get said loan, the man has to choose between an equal annuity (fixed rate) payment method and a decreasing annuity (variable rate) one.
Which of the above two options is financially advantageous?
Resources: The problem, the "Solution" document.
Generalization: Possibilities for changing the problem, changing data.

Another example of activity

Which Car Can I afford?

You want to buy a car. You go to a car dealer and start looking around. One of his first questions that you need to consider is "How much money do you want your monthly payment to be?" You, as the consumer may be thinking that a car loan with a lower monthly payment means that the car is cheaper. There is a variety of loan options available so the length of the loan and the interest rates must also be considered.

Do you think that it is always a good idea to choose a loan with lowest monthly payment? Why is it necessary to consider the length of time that you will be making payments on the loan? Write your response below and justify your reasoning:…….

In order to purchase a car for this activity, you must take out a car loan. You could buy a 2007 Mustang GT Premium Convertible for 31,840 or a 2003 Mustang GT Premium Convertible for 19,120. Which loan offers the better value and is affordable?

2007 Mustang GT Premium, $31, 840 2003 Mustang GT Premium, $19,120

Calculate the missing values for both a 3-year loan and a 5-year loan.

	2007 Mustang	**2003 Mustang**
Table 1.	3-year loan at 6.5%	5-year loan at 7.15%
Number of Months		
Interest (%)		
Principal Value		
Monthly Payment		

5. Real functions of real variable

Field of application: Financial Mathematics
Required knowledge: Affine function and geometric transformation
Project: Real Functions OF Real Variable, 10th grade
Moodle: http://srv-1lyk-aigiou.ach.sch.gr/moodle/course/view.php?id=2
Authors: Marília Tavares de Araújo, Eduarda Rufo e Costa, Diogo Silva
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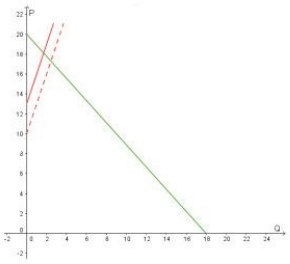
The problem: The coffee plant originating from Ethiopia began to be cultivated around the year AD 575 by the Arabs With the purpose of using it in the preparation of a beverage. The introduction of commercial coffee in Europe took place through Venice, where the first public café "Café Florian" was opened in 1645. Coffee is one of the most popular drinks in Portugal and it is mostly associated with social situations. Assume that the coffee market in Portugal is characterized by demand and supply functions, defined, respectively, by the expressions $P = 20 - 2Q$ and $P = 10 + 3Q$.

Recently, there was an increase in the price of electricity in Portugal, considered as an intermediate good used in coffee production, it is obtained the graphic representation of Picture 1. Look carefully at the new graphic representation and complete the following statements correctly, tracing the expressions that make them incorrect.



Picture 1

Statement I: "The translation of the graphical representation of the supply function associated with the vector $\vec{u}(0,3)$ implies the increase / decrease of the price and / the increase / decrease of the equilibrium quantity.
Statement II: "The increase in the price of electric energy increases / decreases the price of each unit of coffee produced which leads the producer to increase / decrease the price of his products and to an increase / decrease of the amount of equilibrium."
Resources: Worksheet, Geogebra, QR Code, Graphic, Calculator, Tablet, Computer, Kahoot.

Required knowledge and skills: linear functions

Overview

1. Linear function: function whose equation is linear, i.e. has the form $y = mx + b$, where $m \neq 0$
2. Graph of a linear function: straight line
   a. drawing the graph when the equation is given
   b. setting up the equation when the graph is given (see also item 5)
3. Interpretation of the coefficient *m* in the equation $y = mx + b$
   a. slope of the line
   b. rate of change: if *x* increases by 1 unit, then *y* changes by *m* units
   c. difference quotient: $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$, where $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ are points on the line
   d. sign of *m* determines whether line is increasing or decreasing
   e. absolute value of *m* determines how steep the line is
   f. parallel lines have equal slopes
   g. slopes of perpendicular lines have product equal to –1
4. Interpretation of the coefficient *b* in the equation $y = mx + b$
   a. *b* is *y*-intercept
   b. b is function value of 0
   c. $b = 0$ iff origin is on the line
5. Setting up the equation of a line
   a. if slope *m* and one point $P(x_0, y_0)$ are given: $y - y_0 = m(x - x_0)$
   b. if two points are given
6. Linear equations:
   a. solving linear equations
   b. solving other equations leading to a linear equation
   c. (graphical) interpretation of the solution of a linear equation: solution of equation $mx + b = 0$
      i. is *x*-intercept of the corresponding line
      ii. is the zero of the corresponding linear function
7. Linear inequalities
   a. solving linear inequalities
   b. graphical interpretation of linear inequalities

i. solutions of $mx + b > 0$ form interval where corresponding line is above the horizontal axis
ii. solutions of $m_1 x + b_1 > m_2 x + b_2$ form interval where the line corresponding to left hand side is above the line corresponding to right hand side

8. Implicit equation of a line: $ax + by + c = 0$ or, equivalently, $ax + by = d$, (where a and b are not simultaneously 0); horizontal and vertical lines
9. Graphical interpretation of the solution of a system of two linear equations: intersection point of the lines corresponding to the equations
10. Applications and word problems involving linear functions, more specifically: setting up an equation for one or more linear functions given a description in words and using these equations to solve a problem by calculating a function value, solving a linear equation, solving a linear inequality, solving a system of linear equations.

Examples

Example 1. Which of the following functions are linear? Moreover, in case the function is linear, find out whether the function is increasing or decreasing.

A. $y = x^3$
B. $y = 3x$
C. $y = 3x - 5$
D. $y = -5$
E. $y = -2(3x - 5)$
F. $y = x(3x - 5)$
G. $y = x(3x - 5) - 3x^2$
H. $y - 3x = 5$

Example 2. Find the equation of the line through the points $P(2,1)$ and $Q(1,2)$.

Example 3. Find the equation of the line through the point $P(2,1)$ and parallel to the line through $Q(5,2)$ and $R(0,4.5)$.

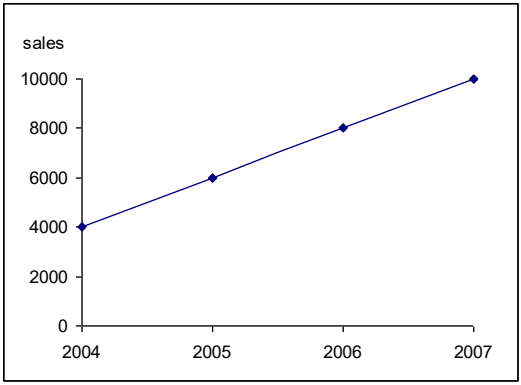
Example 4. Find the equation of the line through the origin and perpendicular to the line having equation $y = 2x + 3$.

Example 5. Are the lines $2x+3y=4$ and $3x-2y=4$ perpendicular to each other?

Example 6. Find the intersection point of the lines $2x+3y=4$ and $3x-2y=4$.

Example 7. The cost of c of producing q units of a certain good consists of two parts. There is a fixed cost of 280 EUR, plus a variable cost of 3 EUR per unit produced. Write an equation giving the cost c in terms of the number q of units produced.

Example 8. A new product was launched in 2004. The graph below shows how many units of this product were sold in the years 2004 to 2007. If the sales continue increasing in the same way, how many units of the product will be sold in 2010?



Example 9. The cost c of a taxi ride of x km is given by the equation $c=5+0.8x$. Make a graph showing the price of taxi rides for a distance between 0 and 10 km.

Example 10. A first electricity company charges a fixed rate of 100 EUR per year and 0.2 EUR per kilowatt-hour of electricity consumed. A second company has no fixed rate, but charges 0.22 EUR per kilowatt-hour consumed. Find out for which yearly consumption of electricity the second company is cheaper than the first one.

Solutions to the examples

Example 1. B (increasing), C (increasing), E (decreasing), G (decreasing), H (increasing)

Example 2. $y=-x+3$

Example 3. $y = -0.5x + 2$

Example 4. $y = -0.5x$

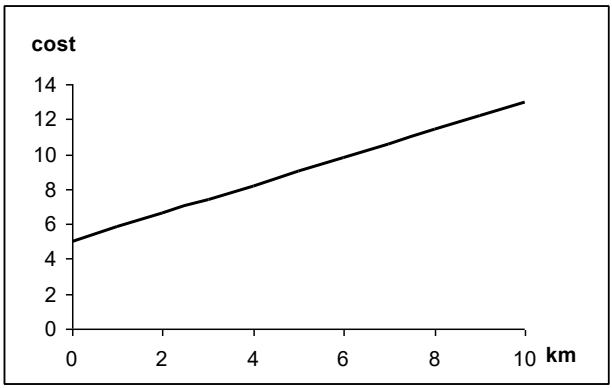
Example 5. yes

Example 6. Intersection point has coordinates $\left(\frac{20}{13}, \frac{4}{13}\right)$

Example 7. $c = 280 + 3q$

Example 8. 16 000

Example 9. see graph below



Example 10. yearly consumption below 5000 kilowatt-hours

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