Erasmus+

Material selected by
Samfirescu Isabela

O2 - Open on-line course topic "Maths in Sports"

TOGETHER
MATHEMATICAL
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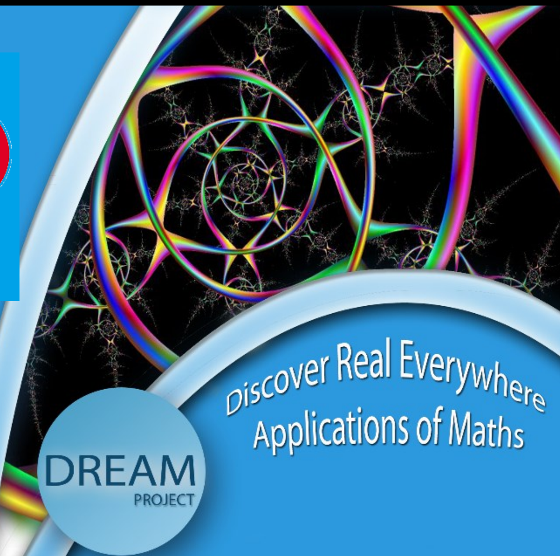
TRUE



Co-funded by ERASMUS+ Program of the European Union, Key Action 2
Project: 2016-1-RO01-KA201-024518"Discover Real Everywhere Applications of Maths – DREAM"
Beneficiary: Colegiul National "Constantin Diaconovici Loga", Timisoara

Timişoara
2018

Colegiul Național ”Constantin Diaconovici Loga” Timișoara, România
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This material is provided with the support of ANPCDEFP through the Erasmus + KA2 program contract no. 2016-1-RO01-KA201-024518, project “DREAM - Discover Real Everywhere Applications of Maths”. The NA and the European Commission are not responsible for the content of this document, and the full responsibility lies with the authors

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Maths in Sport

Contents

Foreword

This intellectual output was create in the Erasmus project " DREAM - Discover Real Everywhere Applications of Maths", identification number: 2016-1-RO01-KA201-024518, through the collaboration of students and teachers from Colegiul Național "Constantin Diaconovici Loga" Timișoara, Romania, 1o Geniko Lykeio, Aigiou, Greece, Agrupamento de Escolas Soares Basto, Oliveira de Azeméis Norte, Portugalia and "TIBISCUS" University of Timișoara, Computers and Applied Computer Science Faculty.

The project main objective was to build up a new maths teaching/learning methodology based on real-life problems and investigations (open-ended math situations), designed by students and teachers together. The activities involved experimentations, hands-on approach, outdoor activities and virtual and mobile software applications. The developed material was transform into on-line courses and is freely available to all interested communities, in order to produce collaborative learning activities.

O2 - Maths in Sport has the purpose to facilitate the understanding of the usefulness of some mathematical chapters that are applicable to Sport and healthy life.

The activities in this pack feed into the Skills and Capability Framework by providing contexts for the development of Thinking, Problem Solving and Decision Making Skills and Managing Information. Open-ended questions facilitate pupils to use Mathematics. ICT opportunities are provided through using Moodle platform and additional tasks researching information using the internet.

This intellectual output comprises five lesson scenarios and guides the teacher in creating interactive and exciting lessons.

Introduction

The importance of a healthy lifestyle

Physical Education class can bring many benefits in understanding mathematics, physics or science in general. Below we will exemplify these things.

Students may find out during the PE class that the dimensions of football fields, volleyball or tennis courts and others are the result of calculating the average distance travelled by the ball (of a certain mass and elasticity) kicked, hit with a racket or thrown.

These dimensions have varied throughout time due to changes in the dimensions or the material from which balls or other sporting goods were made.

Starting from concrete observation of the ball's movement, students can come up with complex problems dealing with throwing objects in a gravitational field, for example, for the Mathematics and Physics classes.

A mathematical space with multiple dimensions is something quite usual for those dealing with pure Mathematics. Human dimensions are easy to be understood as coordinates in a social space with more dimensions. We could use a, b, c, d, e, f, g, h for age, height, mass, gender, shoe size, eye color, hair color, grade etc. In the place of geometrical points we could have people. Limiting ourselves to this 'space' in 8 dimensions, Georgescu Marian would have the following coordinates: 16 years old, 180 cm, 75 kg, male, 43, blue, brown, 10^{th} grade and the coordinates of teacher Popescu Ioana would be: 26 years old, 170 cm, 56 kg, female, 39, hazel, black, teacher. The link between data can be made through correlation and regression.

Correlation measures how close the relationship is between two quantities, for example mass and height, while regression can be used in order to predict the values of one property (mass) with the help of the other (height, in this case).

In Mathematics, starting with Galton, who introduced regression and correlation, all the way to Spearman , who used the correlation of ranks for psychological studies, progress has also been made in problems dealing with sports, like measuring the concord between referees.

A very interesting problem for students, as for the entire society, is the problem of diet. A performance sportsman needs an intake of 120 mg of vitamins and at least 900 mg of minerals each month.

Besides the normal alimentation, to respect such a diet, they use dietary supplements, composed of various percentages of vitamins and minerals. The problem is how many they have to buy each month in order to obtain the necessary quantity of vitamins and minerals. Stigler solved this issue in 1945 by a heuristic method and afterwards by Danzig, in 1947, by linear programming. For students passionate about IT, the problem of diet remains open to the search of new algorithms that can solve the problem of linear programming, like the one found by Karmarkar in 1984. All students were interested by this problem because they, as well as other family members, want to stay fit and healthy.

When the sports class cannot be held in the gym or on the field, students may be captivated by various games. There is only one-step from games to game theory. We start from 2 players, 0 sum and end up with repetitive games, with "WIN-WIN" Mathematics or with the bets that many people are so fond of. Still in the setting of this type of theoretical lessons, touristic orientation may be another option- finding the best route, leading up to solving on the computer for more counties.

Einstein's theory of relativity can be more easily understood by students in concrete situations, for example the practice of different sports: moving their hand with the same motion and in the sense of the ball's movement, their relative velocity is 0 and the ball may be easily caught. Moving the sense in the opposite sense of the ball's movement , the relative velocity of the two is the sum of their velocities and this may be felt in the moment of contact. Understanding these things, students would progress during training, learning how to place and catch the ball, to rotate it, by lateral shots.

Benefits

One of the problems students often face is choosing the right sport to practice, depending on their inborn qualities and their personal data. So by using math to calculate de probabilities at what sport is suited for you, this will not be your problem anymore.
Also by using maths in sports, a person can improve his technique. Is not enought to be a good sportive if you know maths, but also you cannot be the best sportive if you do not.

Being familiar with problems of relativity and kinetic theory, students may avoid accidents. This justifies the use of different kinds of safety equipment in sports with tough contact (football pads and protective gear, hockey helmets, etc.).

On the other hand, by using a little math in sport we captivate the students to use their mind while they exercise. This way, their capacity of thinking in critics moments, their speed at answering a problem and their memory will be improved.

Methodology

Learning and teaching in mathematics can be made more effective where a balance of practical, oral and written tasks is provided. This pack provides information and scenarios to assist in this task. The intention is to provide young people five activities that are related to their age and attainment. One aspect of the pack is the use of the PowerPoint presentations or educational videos in order to stimulate whole-class discussions before and after the activities have been completed. The emphasis should be on helping young people understand what the problems are and to become aware of the technical vocabulary surrounding the issues..

General Pedagogical Recommendations:

- Watching videos which introduces the theme of real-life lesson
- Discovering the link between real life and the mathematical concept that governs the given situation
- Recall theoretical mathematical concepts
- Frontal discussion of the real situation in the matter
- Solving some parts of the problem by group of students using mathematical tools: minicomputers, Geogebra, Excel, internet
- Discussing solutions, looking for the optimal option
- Student's task: loads the optimal solution found on the MOODLE platform
- Teacher’s task: controls the homework of the student and provides a feedback.

Examples from O2 - Maths in Sport use the notions and the properties of following chapters:

- Parabola
- The Calculus behind a Basketball Shot
- Introduction to Probability
- Introduction to Elementary Trigonometry

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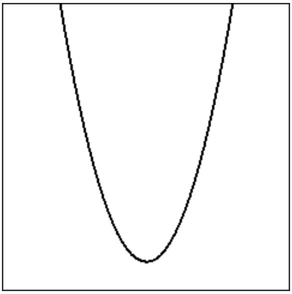
Theoretical background

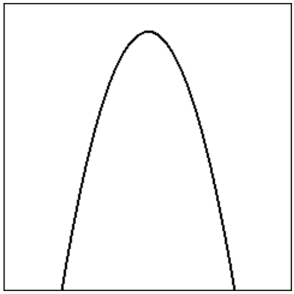
- Parabola

- The graph of any quadratic function $f(x) = ax^2 + bx + c$ is called a *parabola*.
- The graph will have one of two shapes, and the *a* value tells which shape it will be.

graph shape if *a* is positive

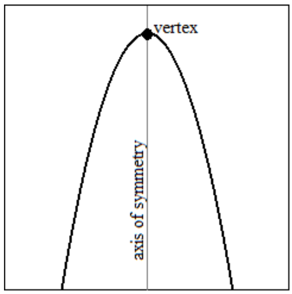


graph shape if *a* is negative



- Every parabola has a special point called the *vertex*. It is the lowest or highest point.

Every parabola is symmetric across a vertical line called the *axis of symmetry*. The vertex is always on this line. The line's equation is x = [the x-coordinate of the vertex].



<u>Finding the vertex by averaging the zeros</u>

Here is how to use a quadratic's zeros to find the coordinates of the vertex:

- First, find the zeros by any method (such as factoring or the Quadratic Formula).
- Find the x-coordinate of the vertex by averaging the zeros (add the zeros then divide by 2).
- Then, you can evaluate $f(x)$ to find out the y-coordinate of the vertex.

Example: Find the vertex and the axis of symmetry of $f(x) = x^2 + 2x - 35$.
Solution:

Factor the function: $f(x) = (x - 5)(x + 7)$.

Then find the zeros: $x = 5,\ x = -7$

x-coordinate of vertex: $x = \dfrac{5+(-7)}{2} = -1$.

y-coordinate of vertex: $y = f(-1) = (-1)^2 + 2(-1) - 35 = 1 - 2 - 35 = -36$.

Answer: The vertex is (–1, –36). The axis of symmetry is the line $x = -1$.

Finding the vertex using formula $x = \dfrac{-b}{2a}$

Here is how to find a quadratic's vertex using a formula.

- The x-coordinate of a parabola's vertex is always $x = \dfrac{-b}{2a}$
- Then, you can evaluate $f(x)$ to find out the y-coordinate of the vertex.
- $y = \frac{-\Delta}{4a};\ \Delta = b^2 - 4ac$

Example: Find the vertex and the axis of symmetry of $f(x) = -3x^2 + 12x + 4$.

Solution: $x = \dfrac{-b}{2a} = \dfrac{-12}{2\,□(-3)} = \dfrac{-12}{-6} = 2$.

$y = f(2) = -3 \cdot 2^2 + 12 \cdot 2 + 4 = -12 + 24 + 4 = 16$.

Answer: The vertex is (2, 16). The axis of symmetry is the line $x = 2$.

- The Calculus behind a Basketball Shot

FINDING THE ARC LENGTH OF A BASKETBALL THROW

The path taken by a basketball when shot can be split into two components, the horizontal (x) direction and the vertical (y) direction. These two components can be represented by the

Parametric equations:

$$x(t) = x_0 + v_0 \cos(\theta)\, t$$

$$y(t) = y_0 + v_0 \sin(\theta)\, t + \frac{1}{2} g t^2$$

The variables are considered to be

x_o is the initial horizontal position of the basketball.

y_o is the initial vertical position of the basketball.

v_o is the initial velocity of the basketball.

θ is the angle the ball is projected with respect to the x-axis.

g is the acceleration due to gravity, -9.81 m/s^2

t is the time traveled.

The derivatives of x(t) and y(t) with respect to time t are:

$$\frac{dx}{dt} = v_0 \cos(\theta)\, t \;\; ; \;\; \frac{dy}{dt} = v_0 \sin(\theta)\, t - 9.81t$$

Now, the distance of the travel distance of the basketball can be found using the arc length equation:

$$L = \int_\alpha^\beta \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt, \;\; \alpha \le t \le \beta.$$

Now, by inserting the derivatives of *x(t)* and *y(t)* in the arc length equation:

$$L = \int_\alpha^\beta \sqrt{(v_0 \cos(\theta)\, t)^2 + (v_0 \sin(\theta)\, t - 9.81t)^2}\, dt.$$

This equation can be modified based on: $(a-b)^2 = a^2 - 2ab + b^2$

$$L = \int_\alpha^\beta \sqrt{v_0^2 cos^2(\theta) + v_0^2 \sin^2(\theta) - 19.62 \cdot t \cdot v_0 \cdot \sin(\theta) + 96.24t^2}\, dt.$$

By further modifying, we can arrive at this formula:

$$L = \int_\alpha^\beta \sqrt{v_0^2 - 19.62 \cdot t \cdot v_0 \cdot \sin(\theta) + 96.24t^2}\, dt.$$

Using the average velocity of a basketball shot, 2.24 m/s, the shot angle that would produce maximum efficiency, 45 degrees, and the time it would take the ball to travel from the free throw line, about 2 s, the arc length can be calculated.

$$L = \int_0^2 \sqrt{2.24^2 - 19.62 \cdot t \cdot 2.24 \cdot \sin(45) + 96.24t^2}\, dt = 17.34.$$

- Introduction to Probability

Experiment

An experiment is a process by which an observation is made; an observation is referred to as an outcome, and an outcome of an experiment cannot be predicted with certainty.

Sample Space

The sample space is the set of all possible outcomes of an experiment.

Event

An event is a set of outcomes of an experiment or a subset of the sample space. Note, a Simple Event or element is an event that cannot be decomposed.

Probability of an Event:

$$P(E) = \frac{\text{number of ways an event can occur}}{\text{number of possible outcomes}} = \frac{n(E)}{n(S)}$$

Union

A and B are two events defined on the sample space S; the union of A and B [AUB] is the event that A occurs or B occurs or both occur.

Intersection

A and B are two events defined on the sample space S; the intersection of A and B [$A \cap B$] is the event that both A and B occur.

Complement of an Event

If E is any event, the event that E does not occur is called the complement of E; it is all the outcomes that are not associated with E but are in the sample space; it is written E' or E^c. Furthermore,

$$P(E') = 1 - P(E)$$

Mutually Exclusive Events

Mutually exclusive events are events that have no outcomes in common.

A probability measure on a sample space S (i.e. collection of all possible outcomes) is a function P from subsets of S to the real numbers that satisfies the following axioms.

Axioms of Probability

1. *Nonnegative:* The probability of an event is a number between 0 and 1.
   $0 \le P(E) \le 1$
2. *Certainty*: The probability of the sample space is 1. $P(S) = 1$
3. *Union*: The probability of the union of mutually exclusive events is the sum of each event's probability.
   $P(AUB) = P(A \text{ or } B) = P(A) + P(B)$

Some Essential Properties of Probability

(Assume A and B are events from the sample space S.)

1. $P(F) = 0$
2. If A is a subset of B, then $P(A) \le P(B)$.
3. $P(E') = 1 - P(E)$
4. **Non Mutually Exclusive Events**: events with common outcomes

If A and B are non-mutually exclusive events,

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = P(A) + P(B) - P(A \cap B)$

Independent/Dependent Events

Two events, A and B, are **independent** if the occurrence of A does not affect the probability of the occurrence of B; otherwise, the events are said to be **dependent**.

If A and B are independent events: $P(A \text{ and } B) = P(A) \times P(B)$

If A and B are dependent events: $P(A \text{ and } B) = P(A) \times P(B/A)$

the occurrence of event B is dependent on the occurrence of event A (written B/A).

<u>Examples</u>

1. Two balls are drawn from a bag containing 3 red, 3 green and 4 black balls. Find the probability that:

   (a) Both are red,
   (b) One is green and one is black,
   (c) Both are of the same color.

(a) $\frac{3}{10}\times\frac{2}{9}=\frac{1}{15}$ **(b)** $\frac{3}{10}\times\frac{4}{9}+\frac{4}{10}\times\frac{3}{9}=\frac{4}{15}$ **(c)** $\frac{3}{10}\times\frac{2}{9}+\frac{3}{10}\times\frac{2}{9}+\frac{4}{10}\times\frac{3}{9}=\frac{4}{15}$

2. One bag contains 4 red balls and 2 blue balls; another bag contains 3 red balls and 5 blue balls. One ball is drawn from each bag, determine the probability that:

(a) Both are red,

(b) One is red and one is blue.

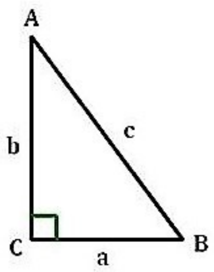
(a) $\frac{4}{6}\times\frac{3}{8}=\frac{1}{4}$ **(b)** $\frac{4}{6}\times\frac{5}{8}+\frac{2}{6}\times\frac{3}{8}=\frac{13}{24}$

- Introduction to Elementary Trigonometry

In a right triangle, the side opposite the right angle is called the hypotenuse, and the other two sides are called its legs. By knowing the lengths of two sides of a right triangle, the length of the third side can be determined by using the Pythagorean Theorem:

Pythagorean Theorem: The Square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of its legs.

Trigonometric Functions of an Acute Angle:



SOH-CAH-TOA

$\sin = \frac{opposite}{hypoteneuse}$	$\sin A = \frac{a}{c}$	$\sin B = \frac{b}{c}$
$\cos = \frac{adjacent}{hypoteneuse}$	$\cos A = \frac{b}{c}$	$\cos B = \frac{a}{c}$
$\tan = \frac{opposite}{adjacent}$	$\tan A = \frac{a}{b}$	$\tan B = \frac{b}{a}$

Trigonometry was originally used to show angles in the sky and the arc of circles; later it was discovered that these functions could be used to determine the sides of triangles. Trigonometric tables were created over two thousand years ago for computations in astronomy.

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Special Angles

Trig Functions of Special Angles (θ)

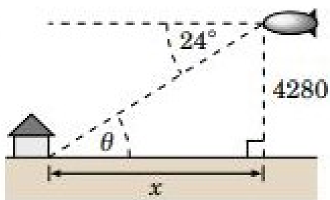
Radians	Degrees	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0°	$\frac{\sqrt{0}}{2}=0$	$\frac{\sqrt{4}}{2}=1$	$\frac{\sqrt{0}}{\sqrt{4}}=0$
$\pi/6$	30°	$\frac{\sqrt{1}}{2}=\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{1}}{\sqrt{3}}=\frac{\sqrt{3}}{3}$
$\pi/4$	45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{\sqrt{2}}=1$
$\pi/3$	60°	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{1}}{2}=\frac{1}{2}$	$\frac{\sqrt{3}}{\sqrt{1}}=\sqrt{3}$
$\pi/2$	90°	$\frac{\sqrt{4}}{2}=1$	$\frac{\sqrt{0}}{2}=0$	*undefined*

Example

A blimp 4280ft above the ground measures an angle of depression of 24^0 from its horizontal line of sight to the base of a house on the ground. Assuming the ground is flat, how far away along the ground is the house from the blimp?

Solution:

Let x be the distance along the ground from the blimp to the house, as in the picture. Since the ground and the blimp's horizontal line of sight are parallel, we know from elementary geometry that the angle of elevation θ from the base of the house to the blimp is equal to the angle of depression from the blimp to the base of the house, i.e. $\theta = 24^0$. Hence,

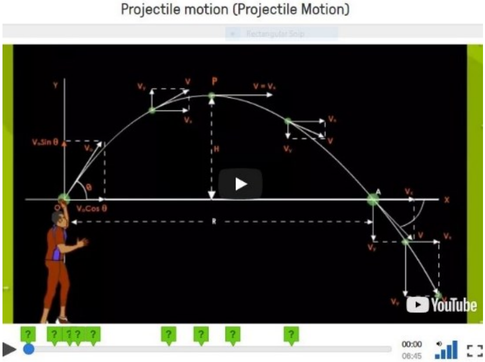


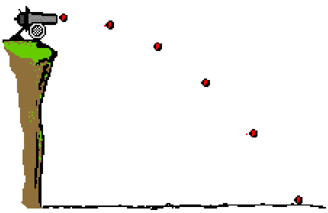
$$\frac{4280}{x} = \tan 24^0 \implies x = \frac{4280}{tan24^0} = 9613\ ft.$$

1. Angle of shot

Field of application: Sports, Mathematics, Physics
Required knowledge: Geometry of angles, Velocity.
Project: Angle of shot
Moodle: http://srv-1lyk-aigiou.ach.sch.gr/moodle/course/view.php?id=6
Authors: Ioannis Andreas Bodiotis, Dimitra Skoura
Coordinator: Nikolaos Diamantopoulos
The assignment: These problems gives students the opportunity to investigate projectile motion and mathematics in a real-life sport context that of shot putter.
1. When a projectile is fired, it travels along a parabola (in the absence of wind and air resistance).A shot putter will release the shot from arm's length. Estimate the optimal angle that the shot should be released from to make it travel furthest, assuming the shot putter can launch the shot at the same speed from any angle. (Note: the shot is launched from around head height rather than ground level)
2. Determine approximately the angle the shot putter should choose, to maximize the length of the shot put.
Resources: video, https://edpuzzle.com/, learning resource http://www.brunel.ac.uk, worksheet, chat, project.
Generalization: Research can be extended to every sports were a ball is thrown

Projectile motion (Projectile Motion)





This projectile is launched with an initial horizontal velocity from an elevated position.

Your goal is to be able to solve problems of range (where the projectile will land), or initial speed (how fast was it going when launched), or height (how far above the target was the projectile when launched. To solve for one of these you must know the other two.

Velocity of shot - What happens in reality?

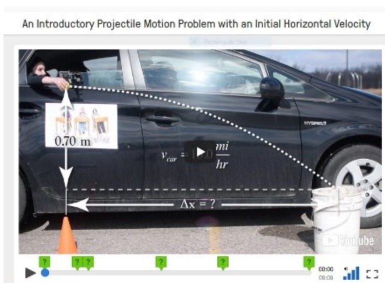
In reality, it is not possible to launch the shot at the same speed from any angle: the body is naturally able to put more power into certain angles. Linthorne (2001) constructed a mathematical model in which the velocity is related to the projection angle as follows (Linthorne has written about the model on the Brunel University site; the published reference is given below)

$$v = \sqrt{\frac{2(F - a\theta)l}{m}}$$

where F is the force (in Newton's) exerted on the shot for a horizontal release angle, a is a constant that characterizes the rate of force decrease with increasing release angle, l is the acceleration path length (in meters) of the shot during the delivery and m is the mass of the shot (7.26kg). A typical set of values for these parameters are F=450N, a=3N/degree, l=1.65m and m=7.26kg.

Bibliography

Linthorne, N. P. (2001). Optimum release angle in the shot put. Journal of Sports Sciences, 19, 359–372.

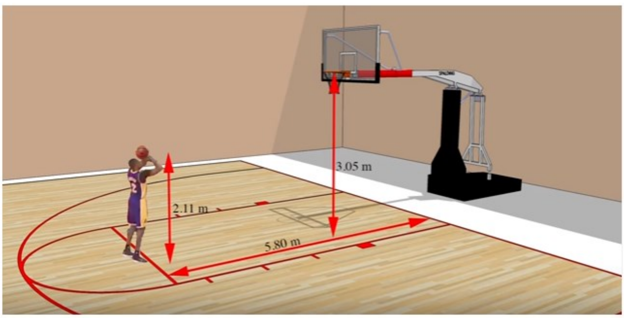


2. Basketball

Field of application: Sports, Mathematics, Physics
Required knowledge: Equation of distance, velocity
Project: Will it make it?
Moodle:http://srv-1lyk-aigiou.ach.sch.gr/moodle/course/view.php?id=6&sesskey=yaLTRMSFY1#section-2
Authors: Aspasia Vogiatzi, Evangelia Karatza
Coordinator: Ilias Spanos
The assignment: Find the perfect basketball free throw
In physics, there are 3 basic equations we can use to help us gather all parts of our situation. Those equations are:

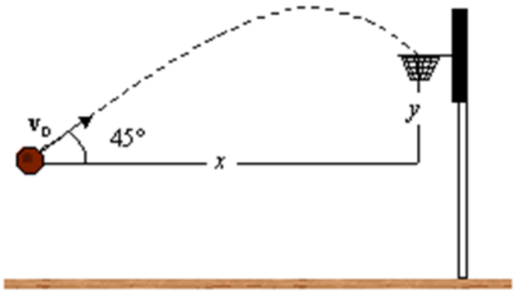
$$v_f = v_i + at$$
$$d = v_i t + \frac{1}{2} a t^2$$
$$v_f^2 = v_i^2 + 2ad$$

Here is a 2.11 m NBA player standing at the foul. We know that foul line is 5.80 m from the basket and we know that basket is 3.05 m from the ground.
Resources: video: GOM player: http://player.gomlab.com/eng/, Geogebra, worksheet
Generalization: Research can be extended to finding the perfect throw to every sport



Example 1

A basketball is launched with an initial speed of 8.0 m/s and follows the trajectory shown. The ball enters the basket 0.96 s after it is launched. What are the distances x and y? **Note**: *The drawing is not to scale*



Answer
Lets take the origin to be where the motion starts as shown.

x – Component	y - Component
$x_0 = 0$	$y_o = 0$
$x = 0$	$y = ?$
$v_{ox} = 8.\text{m/sec} \cos45^o$	$v_{oy} = 8.\text{m/sec} \sin45^o$
$v_x = v_{0x}$	$v_y = ?$
$a_x = 0$	$a_y = -9.8\text{m/sec}^2$
$t = 0.96$ sec	$t = 0.96$ sec

$x = x_o + v_x\, t$
$x = 0 + 5.7\text{m/s}\ (0.96\text{sec})$
$x = 5.43m$
$y = y_o + v_{oy}\, t + ½\, g\, t^2$
$y = 0 + 5.7\text{m/s}(0.96\text{sec}) + ½\ (-9.8\text{m/sec}^2)(0.96\text{sec})^2$
$y = 0.91\text{m}$

Example 2

A tennis ball is shot vertically upward from the surface of an atmosphere-free planet with an initial speed of 20.0 m/s. One second later, the ball has an instantaneous velocity in the upward direction of 15.0 m/s.

A) How long does it take the ball to reach its maximum height?
B) How high does the ball rise?
C) What is the magnitude of the acceleration due to gravity on the surface of this planet?
D) Determine the velocity of the ball when it returns to its original position. **Note**: assume the upward direction is positive.
E) How long has the ball been in the air when it returns to its original position?

Answer

$y_o = 0$

$$v = v_0 + at$$

$$15m/s = 20m/s + a(1.0\sec) \quad --> \quad a = -5.0m/\sec^2$$

$y = ?$ $v_o = 20.\text{m/sec}$

A) At maximum height, v is zero

$v = +15$ m/sec

$$v = v_0 + at$$

$$0 = 20m/s + (-5.0m/\sec^2)t_{\max} \qquad t_{\max} = 4.0\sec$$

$a = ?$ $t = 1.0$ sec

B) $y = y_0 + v_0 t + \frac{1}{2}at^2$

$y_{max} = ?$

$$y_{nax} = 0 + 20m/s(4.0\sec) + \frac{1}{2}(-5.0m/\sec^2)(4.0\sec)^2$$

$t_{max} = ?$ $y_{max} = 40$ *m*

C) $a = 5/\sec^2$ downward

D) In absence of air resistance, a free fall will take same time for each way of the trip, and will have attain same speed on its way down, as it was projected. So:

E) $t_{total} = t_{up} + t_{down} = 4.0$ sec $+ 4.0$ sec $= 8.0$ sec

F) $v_f = v_0 = 20$m/sec but down

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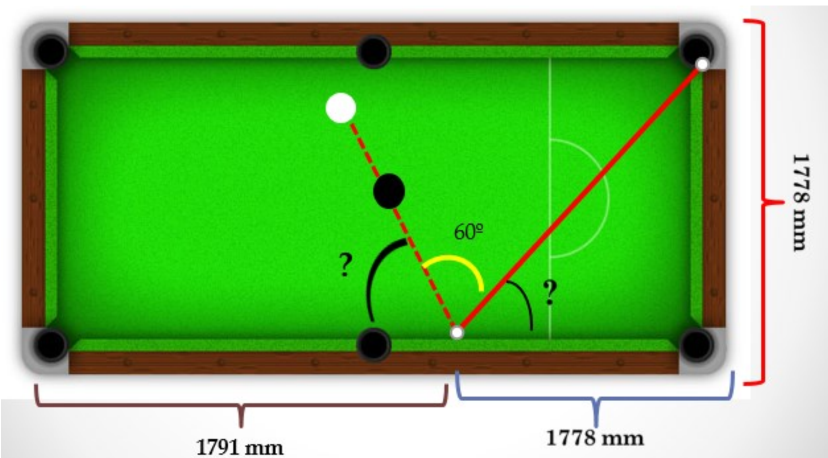
3. Snooker

Field of application: Sports, Mathematics
Required knowledge: Formula of percentage, Surface, Equations, Probabilities
Project: How to be a better player
Moodle: http://srv-1lyk-aigiou.ach.sch.gr/moodle/course/view.php?id=6
Authors: Azgur Ariana
Coordinator: Samfirescu Izabela, Stoia Simona
The assignments:
1. Calculate the angle between the trajectory of the ball and the table, knowing the data's from the picture below.
2. Find the probability of hitting a ball of a certain color.
3. Calculate the price of the tickets knowing that: renting the venue costs 9.500 euro (3.000 places) the players' wages: 40.000 euro, prizes: 30.000 euro, television copyrights 45.000 euro, publicity: 28.000 euro
Resources: Worksheet, project
Generalization: Research can be extended to other sports.



This is a short introduction to the maths of snooker

There are two basic approaches to analyzing snooker mathematically: one is the scoring system and the other is the mechanics of the balls in motion.

The game consists of 22 snooker balls: one white cue ball, 15 red balls and six balls of other colors. Each non-white colored ball corresponds to a number of points when sunk into a pocket, or "potted": 1 for red, 2 yellow, 3 green, 4 brown, 5 blue, 6 pink and 7 black. The highest score one can obtain in a single visit to the table is 147; it is the sum of the points of six red, six black and each of the colored balls.

The physics behind potting the balls.

First, Newton's first law states that a body remains at a constant velocity or at rest unless an external force is applied. Since snooker balls come to rest after being struck, there is indeed an external force acting on it -- the friction on the table. Yes, I mean the soft, grassy table.

Secondly, the friction is directly proportional to the normal reaction (force) on each ball, where the normal reaction is the force the table exerts perpendicularly on the ball due to the latter is weight. Here we note that physicists distinguish between mass and weight: the mass of an object is the amount of matter packed into it, while its weight is the measure of the gravitational pull on it. In other words, the weight of the ball determines the normal reaction "pushing" against it, and the normal reaction on the ball determines the friction acting against the ball. The proportionality constant in the friction-normal reaction relationship is called the coefficient of friction and is determined by experiment.

Thirdly, to know where a ball would end up, we need to find its displacement. According to the equation of motion $v^2 - u^2 = 2as$, where v is the final velocity (0 m/s (meters per second) for a ball at rest), u is the initial velocity (in m/s), a is the acceleration (in m/s^2, meters per second squared) and s is the displacement (in m, meters), we have to find its acceleration and initial velocity from its point at rest. By Newton's second law of motion, we can find the acceleration by dividing the force acting on the ball by the ball's mass. The force on the ball is the difference between the force used to strike the ball and the friction acting on it.

4. Ping-pong

Field of application: Mathematics in Sports
Required knowledge: Formulas of surface, Elementary Geometry
Project: Learning how maths is used in ping-pong
Moodle: http://srv-1lyk-aigiou.ach.sch.gr/moodle/course/view.php?id=6
Authors: Pașca Bianca, Petcu Miruna
Coordinator: Stoia Simona, Samfirescu Izabela
The problems:

1. Your country is hosting the World Championship of Ping Pong. You intend on having five tables in one room, allowing exactly the space needed for a competition of that size for the area around each table.
What are the minimum dimensions of the room you are looking for?

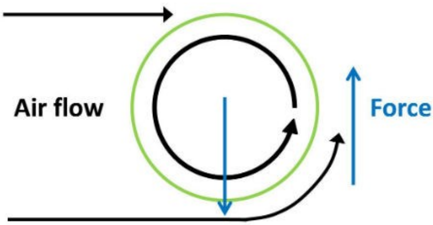
2. You are the server. You imprint with the paddle a velocity of 60 m/s to the ping-pong ball (m=2.7 g), the impact lasting 0.5 seconds. Assuming that the effects of air resistance and gravity are negligible, (the distance is short), calculate the force with which the paddle hits the ball. From what height should you hit the ball in the following figure?

Resources: video, learning resource, worksheet, project.
Generalization: Research can be extended to other sports.

Magnus effect

Magnus effect consists in deflection of the trajectory of a rotating body moving in a gas. It is a direct consequence of the interaction between the body surface and the gas particles.

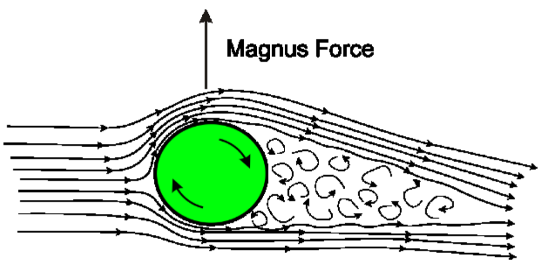
The Magnus effect is why soccer players can bend a soccer ball into the goal around a 5-person wall and why baseball pitchers can throw a breaking ball pitch. A spinning object in motion exerts a net force on the air, which according to Newton's third law exerts an equal and opposite force back on the moving and spinning object, altering its trajectory. As seen in the diagram below, air gets dragged along with the direction of motion, experiencing a (upwards, in this case) force.



It is often referred to in the context of explaining otherwise mysterious but commonly observed movements of spinning balls in sport, especially golf, baseball, football and cricket. Another sport in which the effect is starkly observed is Table Tennis. An experienced player can place a wide array of spins on the ball, the effects of which are an integral part of the sport itself. Table Tennis rackets often have outer layers made of rubber to give the racket maximum grip against the ball to facilitate spinning.

German physicist Heinrich Magnus first described the effect in 1853.

Resource: https://drive.google.com/file/d/1ig-4JdvP6gDM_qnC6L3qmFxnov_U4040/view



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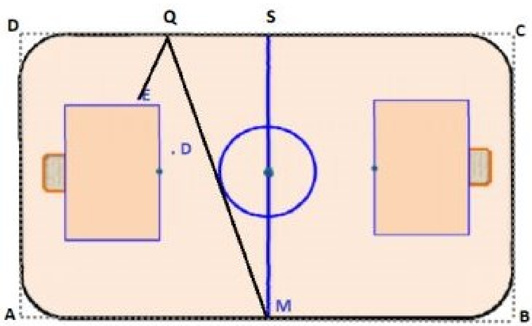
5. Ice Hockey

Field of application: Sports, Mathematics
Required knowledge: Elementary Trigonometry, Geometry of the right triangle
Project: Ice Hockey
Moodle: http://srv-1lyk-aigiou.ach.sch.gr/moodle/course/view.php?id=6&sesskey=oqDEXahnkG#section-5
Authors: Eduarda Costa, Marilia Araujo, Diogo Caetano
Coordinator: Paula Cristina Ornelas
The assignment: The playing court of a roller skating hockey field has a rectangular shape and with proportional dimensions, always respecting the ratio two to one between, respectively, the length and its width. Admit that Figure 1 illustrates the skating field of the União Desportiva Oliveirense (UDO), whose width is 20 m.
In the last class of the subject of Physical Education the teacher of the classes of Eduarda, Marília and Diogo proposed to his students the following situation:
Imagine that Eduarda places herself at point E at 10 m from the [AD] side and at 4 m from the [CD] side, Eduarda's teammate Marília stands at the midpoint on the [AB], M, and Diogo of the opposing team occupies a position, $D\overline{Q}$, between the positions occupied by the opposing colleagues so as not to allow the direct pass between them. Assuming that the angle of incidence of the ball when it hits the lateral fence is equal to the angle of reflection, determine to what distance d of the corner D the ball should hit the lateral fence, so that Marília receives the ball, according to Figure. Present the result with one decimal place.



Resources: video, https://drive.google.com/file/d/131N2X9o_vrw-kflJnHxFcCKYUciVCKsJ/view?usp=sharing, worksheet with problem.
Information source:
http://teachers.henrico.k12.va.us/math/olympics/events/ice_hockey.html
Generalization: Research can be extended to any sports ground.

Hockey problems

1. A team must not have more than six players on the ice while play is in progress. The object is for one team to get the puck (a hard black rubber disc) past the other team's goaltender and into the net, similar to soccer. A regular game consists of three 20-minute periods, with 15-minute intermissions after the first and second periods. If a tie occurs in a game in which a winner must be determined, a sudden-victory overtime period is played. During the gold medal game, a 20-minute, sudden-victory period is played. In the event of a tie after a sudden-victory period, a game-winning shoot-out determines the winner.

The U.S. hockey game started at 6:40 p.m. assuming there were no delays in the 20-minute periods and 15-minute intermissions, what time did the game end?

a) 7:55 p.m

b) 8:10 p.m.

c) 8:25 p.m.

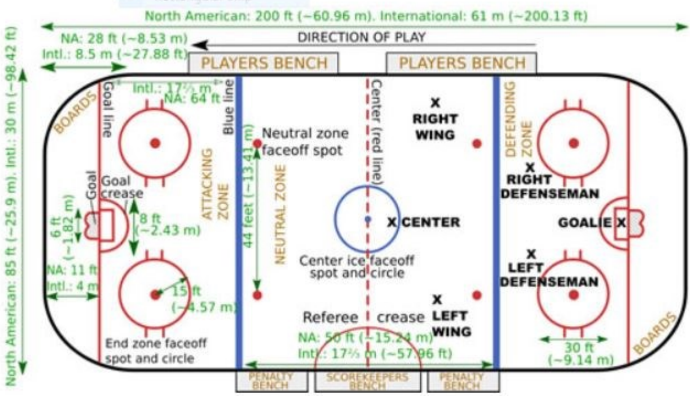
d) 8:30 p.m.

2. In the 2006 Winter Olympics in Italy, the number of teams competing was reduced to 12 and two six-team groups were formed. Each team received 2 points for a win, 0 points for a loss, and 1 point for a tie. The top four teams from each group advanced to the quarterfinals. Below is a table that represents the two groups (A and B) mixed together after the preliminary round.

Based upon the wins/losses/ties, rank the eight teams headed to the quarterfinals from best to worst.

Team	W	L	T	Points
United States	1	3	1	
Russia	4	1	0	
Sweden	3	2	0	
Latvia	0	4	1	
Kazakhstan	1	4	0	
Switzerland	2	1	2	
Canada	3	2	0	
Czech Republic	2	3	0	
Finland	5	0	0	
Italy	0	3	2	
Germany	0	3	2	
Slovakia	5	0	0	

3. There are a lot of measurements when it comes to a hockey rink!



Estimate the total area (in square feet) of the hockey rink (include the corners). Assuming that there are 6 players on the ice, determine the area that each player has to cover provided they are responsible for the same amount of space. Disregard goalies since they are only responsible for the area near the goal.

The team

Colegiul Naţional ”Constantin Diaconovici Loga” Timişoara, România

Fuioagă Gizela Agneta
Patriciu Minerva Floare
Goşa Simona Mihaela
Curea Cristina Marinela
Stoia Simona Laura
Samfirescu Isabela
Schweighoffer Johanna
Ioţcovici Luminiţa
Suciu Ana
Fati Carmen
Neamţu Mihai

Grigoras Flavia
Pasca Bianca
Balica Elena
Lepa Oana
Azgur Ariana
Iorga Patricia
Giurca Cristina
Petcu Miruna
Sbera Alexia
Georgescu Andra

1o Geniko Lykeio, Aigiou, Greece

Efstratios Charitonidis
Nikolaos Diamantopoulos
Ilias Spanos
Spyridon Potamitis
Anastasia Brami

Georgios Kottas
Andreas Bodiotis
Dimitra Skoura
Aspasia Vogiatzi
Evangelia Karatza
Ioannis Papadopoulos
Panagiotis Panagopoulos
Effrosyni-Maria Bouzou
Ilianna Sakellari
Antonia Graikiotou

Agrupamento de Escolas Soares Basto, Oliveira de Azeméis, Portugalia

Maria Cecília De Jesus Oliveira
Maria Emília Castro
Lúcia Maria Azevedo Antão
Paula Cristina Ornelas
Mário Jorge Pina Fonseca Pinto
Maria Virgínia Pinto Feiteira
Ana Luísa Guedes
Margarida Elizabete Costa
Francisco Costa
Jorge Rosa

Marília Tavares Araújo
Margarida Figueiredo De Oliveira
Jacinta Almeida Santos
Eduarda Rufo E Costa
Filipa Sofia Simão Ferreira
Renato Bastos Loureiro
Catarina Mendes Martins Coelho
Mariana Duarte
Diogo Silva
Sílvia Albergaria
Sofia Dias
Jéssica Martins

Universitatea ”Tibiscus”, Timişoara, România

Alexandra Emilia Fortiş
Dan Laurenţiu Lacrămă

Tiberiu Marius Karnyanszky
Pintea Anica Florentina