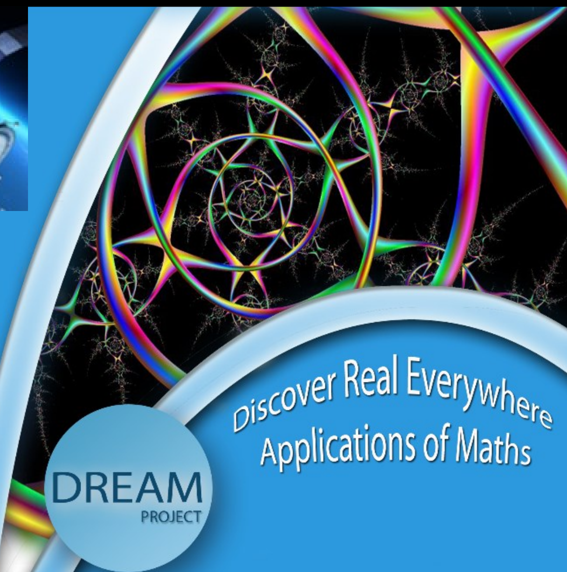
Material selected by
Neamţu Mihai

O3 - Open on-line course topic "Maths in Nature"

TOGETHER
MATHEMATICAL
DREAM
COME
TRUE



Co-funded by ERASMUS+ Program of the European Union, Key Action 2
Project: 2016-1-RO01-KA201-024518"Discover Real Everywhere Applications of Maths – DREAM"
Beneficiary: Colegiul National "Constantin Diaconovici Loga", Timisoara

Timişoara
2018

Colegiul Național ”Constantin Diaconovici Loga” Timișoara, România
1o Geniko Lykeio, Aigiou, Greece
Agrupamento de Escolas Soares Basto, Oliveira de Azeméis Norte, Portugalia
Universitatea ”Tibiscus”, Timișoara, România

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Maths in Nature

Contents

Foreword

This intellectual output was create in the Erasmus project " DREAM - Discover Real Everywhere Applications of Maths", identification number: 2016-1-RO01-KA201-024518, through the collaboration of students and teachers from Colegiul Național "Constantin Diaconovici Loga" Timișoara, Romania, 1o Geniko Lykeio, Aigiou, Greece, Agrupamento de Escolas Soares Basto, Oliveira de Azeméis Norte, Portugalia and "TIBISCUS" University of Timișoara, Computers and Applied Computer Science Faculty.

The project main objective was to build up a new maths teaching/learning methodology based on real-life problems and investigations (open-ended math situations), designed by students and teachers together. The activities involved experimentations, hands-on approach, outdoor activities and virtual and mobile software applications. The developed material was transform into on-line courses and is freely available to all interested communities, in order to produce collaborative learning activities.

O3 - Maths in Nature has the purpose to facilitate the understanding of the usefulness of some mathematical chapters that are applicable in laws of nature.

The activities in this pack feed into the Skills and Capability Framework by providing contexts for the development of Thinking, Problem Solving and Decision Making Skills and Managing Information. Open-ended questions facilitate pupils to use Mathematics. ICT opportunities are provided through using Moodle platform and additional tasks researching information using the internet.

This intellectual output comprises ten lesson scenarios and guides the teacher in creating interactive and exciting lessons.

Introduction

Mathematics exists all around us in the natural world and the teachers have to bring the children's attention to the data and help them see the world around them in a mathematical way.

Mathematical modelling is an important tool used to describe, analyse and predict complex natural phenomenon and processes. Therefore, the environment can be simplified, emulated and understood in an efficient way.

In the teaching process it is important to help pupils to discover and apply knowledge flexibly for studying more effectively. Nowadays, the trends in education highlight the strong connection between mathematical knowledge and the real world applications.

General Pedagogical Recommendations:

– Watching a power point presentation or a film which introduces the theme of real-life lesson
– Discovering the link between real life and the mathematical concept that governs the given situation
– Recall theoretical mathematical concepts
– Frontal discussion of the real situation in the matter
– Solving some parts of the problem by group of students using mathematical tools: minicomputers, geogebra, Excel, internet
– Discussing solutions, looking for the optimal option
– Student's task: loads the optimal solution found on the MOODLE platform
– Teacher's task: controls the home-work of the student and provides a feed-back.

Examples from O3 - Maths in nature use the notions and the properties of following chapters:

– Operations with numbers and percentages;
– First derivative;
– Sequences;
– Fibonacci sequence
– Geometry

Theoretical background

First derivative of a real function

Let $f: D \to R$ be a real function, where D is an interval or an union of intervals within **R**.

Definition 1. We say that function f admits a derivative in $x_0 \in D$ if the limit

$$\lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

exists in $\overline{R}$. In this case the limit is denoted with $f'(x_0)$ and it is called the derivative of the function f with respect to x_0.

Therefore,

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}.$$

Definition 2. Function f is said to be differentiable at $x_0 \in D$ if the limit

$$\lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

exists in R (exists and it is finite). In this case the limit is denoted by $f'(x_0)$ meaning

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}.$$

Observation 1. If function f admits a finite derivative in $x_0 \in D$, then it can be interpreted as the slope of the tangent line at $A = (x_0, f(x_0))$ on the graph of the function f. In this case the equation of the tangent is given by:

$y - f(x_0) = f'(x_0)(x - x_0)$.

Observation 2. If the derivative of function f is infinity the equation of the tangent is given by:

$$x = x_0.$$

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Definition and Examples of Sequences

A sequence is an ordered list of numbers.

$$\left.\begin{array}{l}1,3,5,7,9,\ldots\\-8,3,14,25,\ldots\\1,2,4,8,16,\ldots\end{array}\right\}\text{ are examples of sequences}$$

The three dots mean to continue forward in the pattern established. Each number in the sequence is called a term**.** In the sequence 1, 3, 5, 7, 9, …, 1 is the first term, 3 is the second term, 5 is the third term, and so on. The notation $a_1, a_2, a_3, \ldots a_n$ is used to denote the different terms in a sequence.

The expression a_n is referred to as the **general** or ***n*th term** of the sequence.
Example 1. Write the first five terms of a sequence described by the general term $a_n = 3n + 2$.

$$\begin{aligned}a_n &= 3n+2\\a_1 &= 3(1)+2=5\\a_2 &= 3(2)+2=8\\a_3 &= 3(3)+2=11\\a_4 &= 3(4)+2=14\\a_5 &= 3(5)+2=17\end{aligned}$$

Therefore, the first five terms are 5, 8, 11, 14, and 17.
Example 2. Write the first five terms of $a_n = 2(3^{n-1})$.

$$\begin{aligned}a_n &= 2\left(3^{n-1}\right)\\a_1 &= 2\left(3^{1-1}\right)=2\left(3^0\right)=2(1)=2\\a_2 &= 2\left(3^{2-1}\right)=2\left(3^1\right)=2(3)=6\\a_3 &= 2\left(3^{3-1}\right)=2\left(3^2\right)=2(9)=18\\a_4 &= 2\left(3^{4-1}\right)=2\left(3^3\right)=2(27)=54\\a_5 &= 2\left(3^{5-1}\right)=2\left(3^4\right)=2(81)=162\end{aligned}$$

Therefore, the first five terms are 2, 6, 18, 54, and 162.
Example 3. Find an expression for the *n*th term of each sequence.

1. 2, 4, 6, 8, …
2. 10, 50, 250, 1250, …
3. 3, 7, 11, 15, 19, …

$$2,4,6,8,\ldots \quad \begin{aligned} a_1 &= 2 = 2(1) \\ a_2 &= 4 = 2(2) \\ a_3 &= 6 = 2(3) \\ a_4 &= 8 = 2(4) \end{aligned}$$

1.Based on this pattern, $a_n = 2n$.

$$10,50,250,1250,\ldots \quad \begin{aligned} a_1 &= 10 = 2(5) = 2(5^1) \\ a_2 &= 50 = 2(25) = 2(5^2) \\ a_3 &= 250 = 2(125) = 2(5^3) \\ a_4 &= 1250 = 2(625) = 2(5^4) \end{aligned}$$

2.Based on this pattern, $a_n = 2(5^n)$.

$$3,7,11,19,\ldots \quad \begin{aligned} a_1 &= 3 \\ a_2 &= 7 = 3+4(1) \\ a_3 &= 11 = 3+8 = 3+4(2) \\ a_4 &= 15 = 3+12 = 3+4(3) \\ a_5 &= 19 = 3+16+3+4(4) \end{aligned}$$

3.Based on this pattern,

$$\begin{aligned} a_n &= 3+4(n-1) \\ &= 3+4n-4 \\ &= 4n-1 \end{aligned}$$

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Fibonacci number

Fibonacci posed the following question:

If a pair of rabbits is placed in an enclosed area, how many rabbits will be born there if we assume that every month a pair of rabbits produces another pair, and that rabbits begin to bear young two months after their birth?

This apparently innocent little question has as an answer a certain sequence of numbers, known now as the Fibonacci sequence, which has turned out to be one of the most interesting ever written down. It has been rediscovered in an astonishing variety of forms, in branches of mathematics way beyond simple arithmetic. Its method of development has led to far-reaching applications in mathematics and computer science.

But even more fascinating is the surprising appearance of Fibonacci numbers, and their relative ratios, in arenas far removed from the logical structure of mathematics: in Nature and in Art, in classical theories of beauty and proportion.

Consider an elementary example of geometric growth - asexual reproduction, like that of the amoeba. Each organism splits into two after an interval of maturation time characteristic of the species. This interval varies randomly but within a certain range according to external conditions, like temperature, availability of nutrients and so on. We can imagine a simplified model where, under perfect conditions, all amoebae split after the same time period of growth.

So, one amoebas becomes two, two become 4, then 8, 16, 32, and so on.

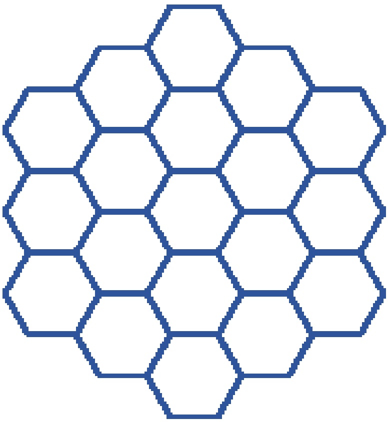
The pattern we see here is that each cohort or generation remains as part of the next, and in addition, each grown-up pair contributes a baby pair. The number of such baby pairs matches the total number of pairs in the previous generation. Symbolically

- f_n = number of pairs during month n
- $f_n = f_{n-1} + f_{n-2}$

So we have a recursive formula where each generation is defined in terms of the previous two generations. Using this approach, we can successively calculate fn for as many generations as we like.

This sequence of numbers 1,1,2,3,5,8,13,21,... and the recursive way of constructing it ad infinitum, is the solution to the Fibonacci puzzle.

Regular Hexagons



Hexagons are interesting geometric shapes and appear in many areas of nature and life.

A hexagon is a 6-sided, 2-dimensional geometric figure. All of the sides of a hexagon are straight, not curved. Hexagons are found in honeycombs created by bees to store honey, pollen, and larvae. They're even famously found in the interlocking columns of volcanic rock that form the Giant's Causeway in Ireland. While these examples might be the most well-known, hexagons are found in many other parts of nature: the bond-shapes of certain molecules, in crystal structures, in the patterns of turtle shells, and more. One place that hexagons occur in nature is in water ice: the molecules of water (two hydrogen atoms and one oxygen atom) always freeze together in the shape of hexagons. We also use hexagons as one way to prove that the circumference of a circle is $2\pi r$.

A hexagon is a flat shape, all in one plane, with six sides all of equal length. Each of the six angles measures 120 degrees, so the total interior angles of a hexagon measure 720 degrees (120 multiplied by 6).

The hexagon can be cut into six equilateral triangles, all the same size.

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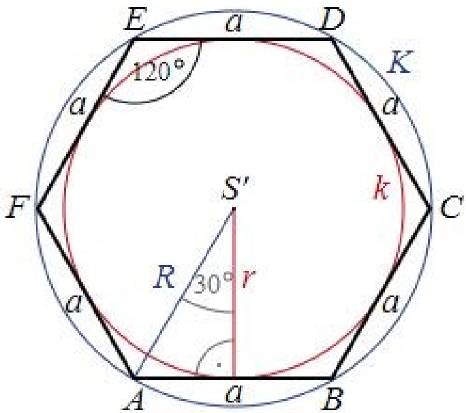


$$A = \frac{3 \cdot \sqrt{3}}{2} a^2$$

$$P = 6 \cdot a$$

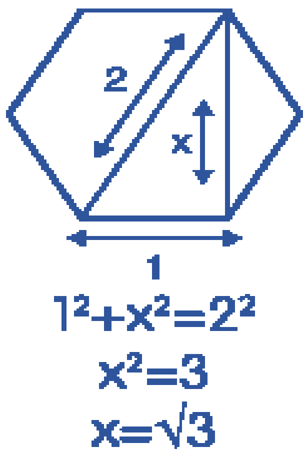
$$R = a$$

$$r = \frac{a \cdot \sqrt{3}}{2}$$

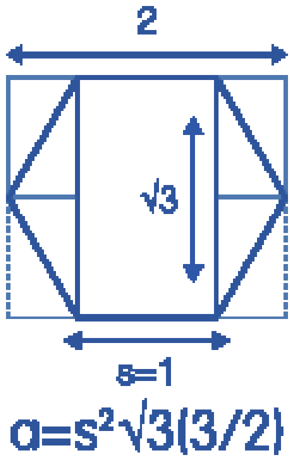
<u>The perimeter</u> of a hexagon is the sum of the lengths of all the sides. All the sides are the same length, so that's the same as the length of one side multiplied by six

<u>The area of a hexagon</u> is the same as the areas of all six equilateral triangles added together (or one equilateral triangle multiplied by six).
[https://en.wikipedia.org/wiki/Hexagon]

<u>The apothem of a regular hexagon</u> is equal to half the square root of 3. This can be demonstrated with a unit hexagon, where each side is given the length of 1. We can then draw a right triangle using the vertex-to-vertex diameter of the hexagon as a hypotenuse. Since the diameter of such a hexagon is known to be 2, we are left with one undefined edge of our triangle, equal the distance between two opposing edges of the hexagon (which is twice the apothem).



Let us call this edge x. The Pythagorean Theorem then tells us that $1^2+x^2 = 2^2$. It then follows that $1 + x^2 = 4$, $x^2 = 3$, and thus $x = \sqrt{3}$. Therefore, the "height" to "width" of a regular hexagon is equal to $\sqrt{3}/2$. Quod erat demonstrandum.



<u>Hexagon Diagonals</u>

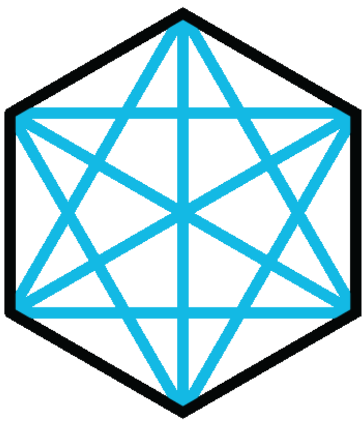
A diagonal is a line segment that connects non-adjacent vertices in a polygon.

The formula to find diagonals of an n-sided polygon is given by

Number of diagonals = $n(n-3)/2$

For hexagon, $n = 6$

=> Number of diagonals = $6(6-3)/2 = 9$.

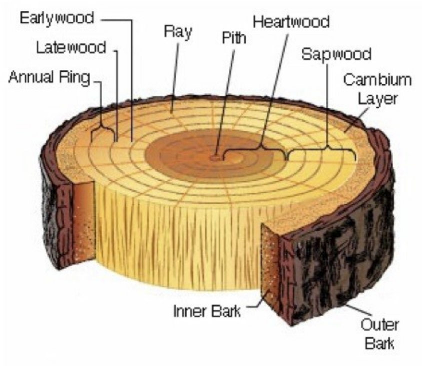


<u>A Tessellation (or Tiling)</u> is when we cover a surface with a pattern of flat shapes so that there are no overlaps or gaps.

Create Hexagon Tessellation!

1. The age of a tree

Field of application: Mathematics in nature
Required knowledge: Calculations with real numbers, circumference, diameter of cycle, pi
Project: Determine the Age of a Tree
Moodle: http://srv-1lyk-aigiou.ach.sch.gr/moodle/course/view.php?id=4
Authors: Georgios Kottas, Andreas Bodiotis, students from 1o Geniko Lykeio, Aigiou, Greece
Coordinator: Spyridon Potamitis
The problem: How old is the giant tree in your backyard? If you don't know the date the tree was planted, there are two main ways you can figure out how old it is. You can either cut it down or use an increment borer to count the tree's rings, or multiply the tree's diameter and the growth factor to estimate the age.
The assignment: Select a tree and hypothesize about the age of the tree based on its size. Write down your predictions so you can compare these to the estimated age you determine for your tree after you complete your measuring. Measure the circumference of the trunk. Use the trunk width at 1.4 m off the ground.
Calculate the diameter. How?
Multiple the diameter and the tree species' average growth factor. This will give you the approximate age of the tree in years. You can figure out a tree's growth factor by searching online.



Submit a file with video/image proofs of your task that also describes the mathematical issues involved.
Resourses: Worksheet

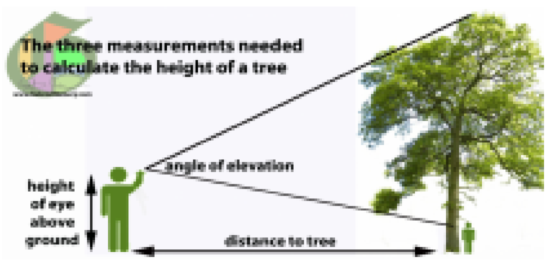
2. Installation of a fast satellite internet in Colegiul National C. D. Loga

Field of application: Mathematics in nature
Required knowledge: Geometry, circumcenter of triangle
Project: Installation of a fast satellite internet in Colegiul National Constantin Diaconovici Loga
Moodle: http://srv-1lyk-aigiou.ach.sch.gr/moodle/course/view.php?id=4
Authors: Dimitra Skoura, Evangelia Karatza students from 1o Geniko Lykeio, Aigiou, Greece
Coordinator: Ilias Spanos
The problem: Your school, Colegiul Naţional Constantin Diaconovici Loga has reached an agreement with two other neighboring institutions, the Colegiul Tehnic Emanuil Ungureanu and the West University of Timişoara to install a fast satellite internet.
For reasons of fairness, installation of satellite antenna should be equidistant from the 3 educational institutions.
For installation of the satellite dish will need to get permission from the city council of Timisoara. The technical service of the city council requires you to identify the exact point with geographic coordinates (longitude, latitude) where the antenna should be mounted .
Can you submit to the technicians of city council a map with the exact stigma of the installation location in order for them to proceed your request?
Describe the geometrical process you followed to answer.
The assignment of this lesson is to submit a file to describe the geometrical process you followed to answer.
Resourses: File: "Assignment" , worksheet

3. Measure of the height of a high object

Field of application: Mathematics in nature
Required knowledge: Calculations with numbers
Project: Measure the height of a high object
Moodle: http://srv-1lyk-aigiou.ach.sch.gr/moodle/course/view.php?id=4
Authors: Andreas Bodiotis, Georgios Kottas students from 1o Geniko Lykeio, Aigiou, Greece
Coordinator: Nikolaos Diamantopoulos
The problem: A clinometer, also called a declinometer or an inclinometer, is an instrument that measures vertical slope, usually the angle between the ground or the observer and a tall object. A simple, or *fixed angle*, clinometer requires plenty of room to walk back and forth when measuring object. A *protractor clinometer* lets you measure while standing in place, and is an easy-to-make version of the clinometers frequently used in astronomy, surveying, engineering, and forestry. Construct one fixed angle clinometer 45^o and one protractor clinometer.
The assignment:
1. Construction of clinometers
2. Recording the measurements
3. Visualize your measurements
4. Calculate the height of your school

Resourses: Files: "Construction of clinometers", "Recording of measurements", "Visualize your measurements", "Calculate the height of your school"



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4. Nearest shelter in the forest

Field of application: Mathematics in nature
Required knowledge: Euclidean geometry, Geogebra or Sketchpad, perpendicular bisector, topology
Project: Nearest shelter in the forest
Moodle: http://srv-1lyk-aigiou.ach.sch.gr/moodle/course/view.php?id=4
Authors: Evangelia Karatza, Aspasia Vogiatzi students from 1o Geniko Lykeio, Aigiou, Greece
Coordinator: Spyridon Potamitis
The problem: Suppose that in a forest there are some built shelters for visitors and holidaymakers. These shelters are intended to offer shelter and food to someone who is found in an emergency during a visit to the forest. Your job is to construct a map which will indicate to excursionists the nearest shelter that can appeal in case something happens. Figures 1 and 2 show the map of the forest with shelters and a simplified form, respectively.

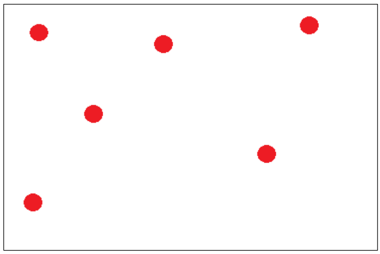
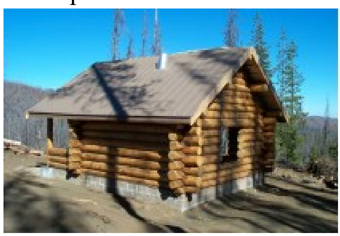


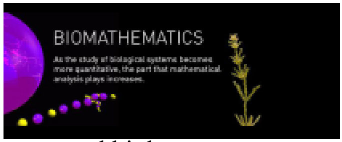
Figure 1: Forest area with shelters be marked in red
Figure 2: Simplified representation of shelters

Following the same logic that you saw earlier, select the map of a forest of your country and add 5 shelters. Submit the final map with the solution of the problem as an image file.



The assignment of this lesson is to submit a file with the solution.
Resourses: Files: "The problem", "To solve the problem", "Assignment", "Question to investigate" , Presentation, worksheet.

5. How mathematics can be applied in nature and biology



Field of application: Mathematics in nature, Biomathematics
Required knowledge: Calculations with numbers, percentages
Project: How mathematics can be applied in nature and biology

Moodle: srv-1lyk-aigiou.ach.sch.gr/moodle/course/view.php?id=4

Authors: Giurca Lucia student from Colegiul Naţional Constantin Diaconovici Loga, timisoara, Romania
Coordinator: Schweighoffer Johanna

The problem:

1. Calculate the total blood mass of a man whose weight is 90 kg and the blood mass of a student who has 45 kg.
2. One adult who has 70 kg loses 30% of his blood because of an accident he had suffered. Calculate the total mass of blood cells themselves he has after the hemoragy.
3. After she had suffered an accident, a woman who has 90 kg loses 0,5L of blood and goes to the hospital for a transfusion. Calculate the total water mass in her blood after the hemoragy.

Maths is often used in nature, for example: biology analysis, calculating the birthrate of some species (ex: rabbits), calculating the wood volume.

The human blood contains: PLASMA (60% of the total volume), BLOOD CELLS THEMSELVES (40% of the blood volume), The total mass of blood represents 8% of the body weight, 90 % of the total plasma is water.

Also, we need to know that when we try to calculate the blood volume, we consider 1liter as being equal to 1 kilogram (1L=1kg).

The AB0 system:

- 0I donates to all groups;
- AII interacts with AII, ABIV and OI;
- BIII interacts with BIII, ABIV and OI;
- ABIV can receive blood from any group.

Example. After he has suffered an accident, a man whose weight is 65 kg, lost 20% of his blood.

a) Calculate the total mass of water which can be found in his circulatory system;

b) The quantity of blood cells he needs in order to survive.

We consider the following: G as being the initial weight, V as being the blood volume, P as being the plasma volume, A as being the water volume out of plasma.

Useful formulas: V=G*0,08, P=0,6*V=0,048*G, A= 0,1*P=0,0048*G. We have:

mass=65kg;

8/100x65=5,2 L of blood before the loss;

He loses 20% of 5,2L, so that's 20/100x5,2=1,04;

He has now only 5,2-1,04=4,16L of blood;

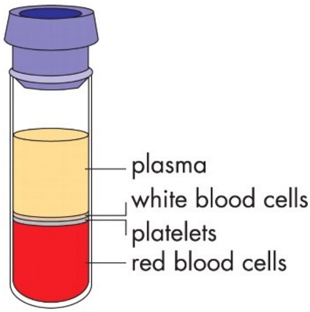
60% of this blood is PLASMA, so 60/100x4,16L=2,496L of blood plasma, of which 90/100x2,496=2,2464L is the water which can be found in his circulatory system.

b) mass=65kg;

He loses 20% of his blood, which is 1,04L of which 40/100x1,04L =0,416L are the blood celles themselves which he need to survive.

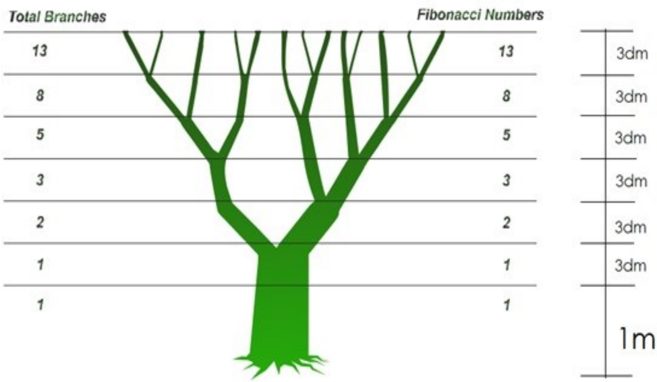
Functions can be really useful in many other circumstances, for instance: calculating the wood volume for a given period of time.

The assignment of this lesson is to submit the solution
Resourses: Files: "Presentation", "Assignment"



6. Fibonnacci

Field of application: Mathematics in nature
Required knowledge: Calculations w
numbers
Project: Fibonnacci
Moodle: http://srv-1lyk-aigiou.ach.sch.gr/moodle/course/view.php?id=4&sesskey=mftwkeCSgG#section-6
Authors: Team of Portugal, Agrupamento de Escolas Soares Basto, Oliveira de Azeméis
Coordinator: Paula Cristina Sousa Pereira Ornelas
The problem: *How high is a tree for it to have 34 branches?*
In a tree, the number of branches is related with its high according to the Fibonacci sequence.



The assignment of this lesson is to answer the question.
Resourses: Files: "Fibonacci theory", "Worksheet"

7. Pollution

Field of application: Mathematics in nature
Required knowledge: Calculations with numbers, percentages
Project: Pollution
Moodle: http://srv-1lyk-aigiou.ach.sch.gr/moodle/course/view.php?id=4&sesskey=mftwkeCSgG#section-8
Authors: Team of Portugal, Agrupamento de Escolas Soares Basto, Oliveira de Azeméis
Coordinator: Paula Cristina Sousa Pereira Ornelas
The problem: $CO_2 + H_2O \dashrightarrow C_6H_{12}O_6 + O_2$
Answer to the questions related to the issue of pollution:

a) Write the stecheometrical coefficients on the chemical equation.
b) Considering that, in average, one house releases 0,825 ton of CO_2 to the atmosphere, state the chemical quantity of the molecule that is released.
c) A tree absorbs 165 Kg of CO_2. What is the quantity of oxygen produced through photosynthesis. Write the answer in Kg and in number of molecules (N_A=6.023x10^{23} mol^{-1}).
d) In "Área Metropolitana do Porto", in Portugal, there is the project of "The 100 000 trees". These trees are enough to cancel the effect of how many houses? And how many factories?
e) Considering that, for a specific type of trees (pine tree), the space between them should be of 2,5m, how many trees could be planted on a plot of 100 hectares?
f) There are 45 508 companies listed in stock exchanges around the world. How many trees would you need to absorb their CO_2 production?

The assignment of this lesson is to submit the answers of the questions.
Resourses: Files: "Air pollution theory", "Worksheet - Photosynthesis"

8. Numbers in nature

Field of application: Mathematics in nature
Required knowledge: Calculations with numbers, Fibonacci sequence
Project: Numbers in nature
Moodle: http://srv-1lyk-aigiou.ach.sch.gr/moodle/course/view.php?id=4&sesskey=mftwkeCSgG#section-8
Authors: Dimitra Skoura, Evangelia Karatzastudents from 1o Geniko Lykeio, Aigiou, Greece
Coordinator: Nikolaos Diamantopoulos
Classroom Activity Sheet: Below are examples from nature in which Fibonacci numbers can be found. Using the illustrations provided below, work with your group to answer the questions.
Flower petals: Count the number of petals on each of these flowers. What numbers do you get? Are these Fibonacci numbers?
Seedheads: Each circle on the enlarged illustration represents a seed head. Look closely at the illustration. Do you see how the circles form spirals? Start from the center, which is marked in black. Find a spiral going toward the right. How many seed heads can you count in that spiral? Now find a spiral going toward the left. How many seed heads can you count there? Are they Fibonacci numbers?
Cauliflower florets: Locate the center of the head of cauliflower. Count the number of florets that make up a spiral going toward the right. Then count the number of florets that make up a spiral going toward the left. Are the numbers of florets that make up each spiral Fibonacci numbers?
Apple: How many points do you see on the "star"? Is this a Fibonacci number? What shape emerges most often from the Fibonacci numbers? What function do you think this shape serves?

The assignment of this lesson is to submit the answer it in the Moodle platform.
Homework: Creating the Fibonacci Spiral
Resourses: Chat, worksheets

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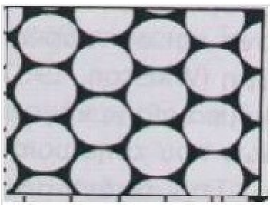
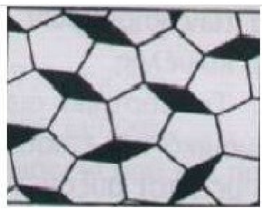
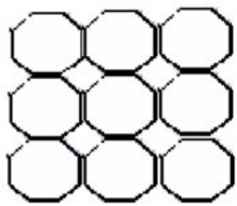
9. Honeycombs of bees – Regular polygons

Field of application: Mathematics in nature
Required knowledge: Polygons
Project: Honeycombs of bees – Regular polygons
Moodle: http://srv-1lyk-aigiou.ach.sch.gr/moodle/course/view.php?id=4&sesskey=mftwkeCSgG#section-9
Authors: Andreas Bodiotis, Georgios Kottas from 1o Geniko Lykeio, Aigiou, Greece
Coordinator: Nikolaos Diamantopoulos
The problem: Bees have been observed to build hexagonal cells in their honeycombs. But where is this persistent repetitive form of cells due?
If the shape of the cells were circular, octagonal or pentagonal then it would not fill all available space, as the corners that are joined should have sum 360 degrees, so there would be gaps and the walls should have been double, resulting in waste of time and material.







The only regular polygons whose angles are divisors of 360 are:

The equilateral triangle (angle: 60 °), the square (angle: 90 °) and the regular hexagon (angle: 120 °). Nevertheless, bees choose the regular hexagon, because it has the largest surface in relation to its perimeter. Can you show it mathematically?
The assignment of this lesson is to submit the answer.
Resourses: Files: "Assignment – Are bees architects that know math?"

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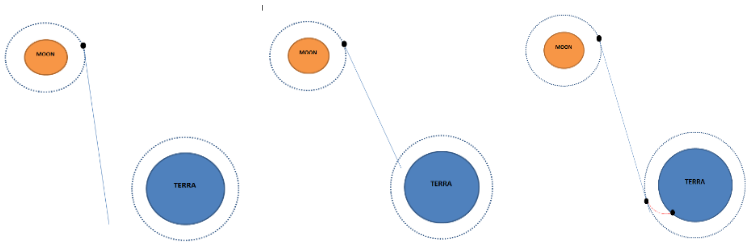
10. Derivative application in navigation into outerspace

Field of application: Mathematics in nature
Required knowledge: First derivative, Geogebra
Project: Derivative application in navigation into outerspace
Moodle http://srv-1lyk-aigiou.ach.sch.gr/moodle/mod/resource/view.php?id=314&forceview=1
Authors: Lepa Oana, Balica Elena students from Colegiul Naţional Constantin Diaconovici Loga, Timisoara, Romania
Coordinator: Neamţu Mihai
The problem: Students will understand the geometrical meaning of the first derivative and will be able to calculate the derivative of a function in a given point.



We suppose the equation of the Earth's atmospheric shape is given by the function

$$f:[-5,5]\to R,\ f(x)=\sqrt{7-\frac{x^2}{3}}\ .$$

Based on the above function, in what follows, using GeoGebra we want to visualize three cases of the spaceship's trajectory, where the gravitational force is low, high and optimal, respectively.

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Let us have the spaceship's position defined by S having the coordinates as (5,5).

In the first the case we do not have a safe landing due to a low gravitational force that will result into a ricochet in outer space of the spaceship. The trajectory of the spaceship will intersect the point with the coordinates A=(-4,2) and the equation is given by:

$$x-3y+10=0.$$

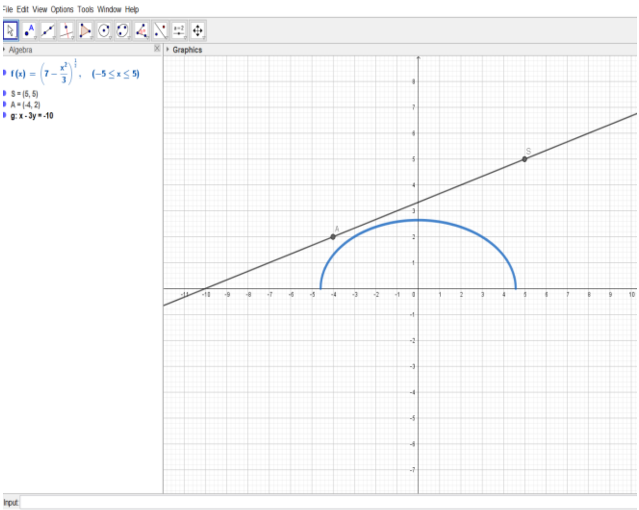

Fig. 1. Outerspace trajectory

In the second case, we do not have a safe landing due to high gravitational force that will result into a descending speed that generates heat above the heat shield resistance limit.

The trajectory of the spaceship will intersect the point with the coordinates A=(-2, 1) and the equation is given by:

$$4x-7y+15=0.$$

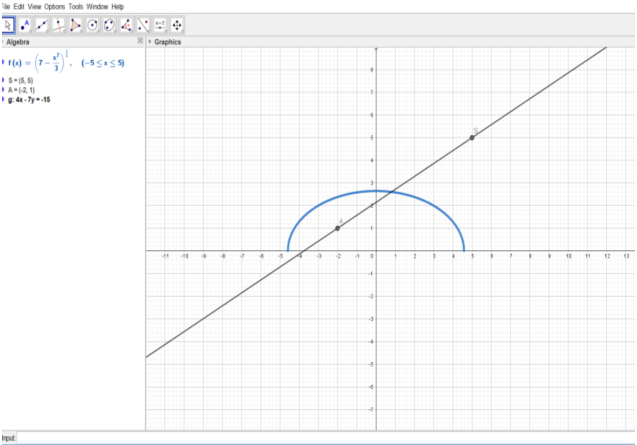

Fig. 2. Crash trajectory

In the third case, in order to assure a safe landing the option is to follow a trajectory which is tangent to Earth's atmosphere. If we choose to reach the point with the coordinates A=(-2.5, 2.2), the equation of the tangent of f at the point A is given by:

$$y - f(-2.5) = f'(-2.5)(x + 2.5)$$

or equivalently

$$y = 0.38x + 3.16.$$

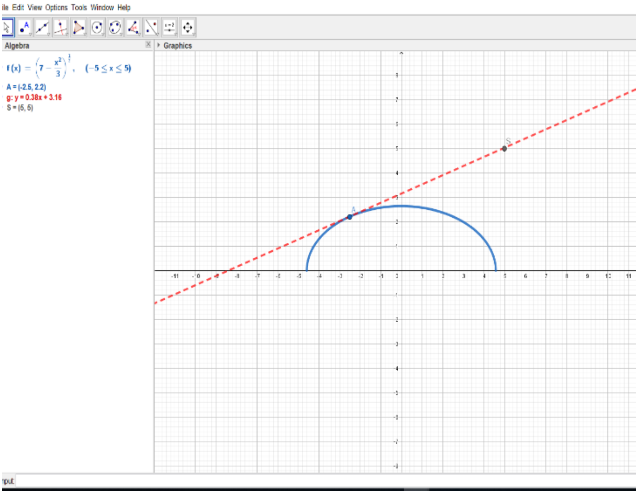

Fig. 3. Optimal trajectory

The assignment of this lesson is to submit a file with the answer
Resourses: Files: "Presentation", "The derivative animation in Geogebra", "Apollo 13 mission - synopsis", "Apollo 13 movie trailer", "Practical assignment", "Supporting videos"

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